18.06 Problem Set 5

Due Wednesday, March 21, 2007 at **4:00 p.m.** in 2-106

Problem 1 *Wednesday 3/14*

Do problem 10 of section 4.3 in your book.

Problem 2 *Wednesday 3/14*

Do problem 17 of section 4.3 in your book.

Problem 3 *Wednesday 3/14*

Find a function of the form $f(t) = Csin(t) + Dcos(t)$ that approximates the three points $(0, 0)$, $(\pi/2, 2)$ and $(\pi, 1)$. In other words, find coefficients C and D such that the error $|f(0) - 0|^2 + |f(\pi/2) - 2|^2 + |f(\pi) - 1|^2$ is as small as possible.

Problem 4 *Wednesday 3/14*

The `MATLAB` command `a=ones(n,1)` produces and n-by-1 matrix of ones. The command `r=(1:n)'` gives the vector $(1, 2, \ldots, n)$ transposed to a column by '. The command `s=r.^3` gives the column vector $(1^3, 2^3, \ldots, n^3)$, because the dots mens "a component at a time."
The purpose of this problem is to find the line $y = c + dt$ closest to the cubic function $y = t^3$ on the interval $t = 0$ to $t = 1$.
(a) Find the best line using calculus, not `MATLAB`. Choose c and d to minimize

$$E(c, d) := \int_0^1 (c + dt - t^3)^2 dt$$

(Hint: find $E(c, d)$ in terms of c and d, and use any method learned in 18.02 to minimize this.)

(b) With $n = 15$, choose C and D to give the line $y = C + Dt$ that is closest to t^3 at the points $t = \frac{1}{15}, \frac{2}{15} \ldots, 1$. Use `MATLAB` to do least squares in order to find C and D, and the differences $c - C$ and $d - D$.

(Hint: Set up the equations you want to solve. Your equations (in matrix form) should involve the vectors `a`, `r/n` and `s/n^3`).

(c) Repeat for $n = 30$. (Notice how `r/n` and `s/n^3` end at 1). Are the differences $c - C$ and $d - D$ smaller for $n = 30$? By what factor?

Problem 5 *Friday 3/16*

Consider in $\mathbb{R}^4$ the subspace given by $F = \{(x, y, z, w) : -x + y + 2z - w = 0\}$.

(a) Give a basis for F.
(b) Use Gram-Schmidt to transform your basis into an orthonormal basis.
(c) What is the distance between the point (1, 3, 1, 1) and (the closest point to) F?

Problem 6 *Friday 3/16*

Do problem 6 of section 4.4 in your book.

Problem 7 *Friday 3/16*

(a) Find a 3-by-3 orthogonal matrix A such that
$A\begin{bmatrix}1\\0\\0\end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\-1\end{bmatrix}$.
(b) How many matrices A are there that satisfy these conditions?

Problem 8 *Monday 3/19*

Do problem 9 of section 5.1 in your book.

In the following problems explain how you calculated the determinants.

Problem 9 *Monday 3/19*

Calculate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}.$$

Problem 10 *Monday 3/19*

(a) Calculate the determinants of the following "almost upper-triangular" matrices:
$A_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$,
$A_3 = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$,
$A_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$,
$A_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$.
(b) Can you figure out how to continue the sequence of matrices $A_2, A_3, A_4, A_5, \ldots$ and calculate $det(A_n)$ for any n?