18.06 Spring 2006 - Problem Set 5

SOLUTIONS TO SELECTED PROBLEMS

1. Section 4.3, Problem 12

Answer:

a) $\mathbf{a}^T\mathbf{a} = m$, $\mathbf{a}^T\mathbf{b} = b_1 + \ldots + b_m$. Therefore $\mathbf{a}^T\mathbf{a}\hat{\mathbf{x}} = m\hat{\mathbf{x}} = b_1 + \ldots + b_m$ and $\hat{\mathbf{x}}$ is the mean of the b's.

b) $\mathbf{e} = \mathbf{b} - \hat{\mathbf{x}}\mathbf{a}$, $\|\mathbf{e}\|^2 = \sum_{i=1}^{m}(b_i - \hat{\mathbf{x}})^2$.

c) $\mathbf{p} = (3,3,3)$, $\mathbf{e} = (-2,-1,3)$, $\mathbf{p}^T\mathbf{e} = 0$. $P = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

2. Section 4.3, Problem 17

Answer:

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 21 \end{bmatrix}.$$

The solution $\hat{\mathbf{x}} \begin{bmatrix} 9 \\ 4 \end{bmatrix}$ comes from $\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 35 \\ 42 \end{bmatrix}$.

3. Section 4.3, Problem 27

Answer: $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 4 \end{bmatrix}$ has $A^TA = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$,

$$A^T\mathbf{b} = \begin{bmatrix} 8 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ \frac{-3}{2} \end{bmatrix}.$$

At $(x, y) = (0, 0)$, the best plane $2 - x - \frac{3}{2}y$ has height $C = 2$ which is the average of $0, 1, 3, 4$.

4. Section 4.4, Problem 7

Answer: If Q has orthonormal columns the least squares solution to $Q^T Q\hat{\mathbf{x}} = Q^T\mathbf{b}$ is $\hat{\mathbf{x}} = Q^T\mathbf{b}$.

5. Section 4.4, Problem 24

Answer:

a) One basis for $\mathbf{S}$ is $\mathbf{v}_1 = (1, -1, 0, 0)$, $\mathbf{v}_2 = (1, 0, -1, 0)$, $\mathbf{v}_3 = (1, 0, 0, 1)$.

b) A basis for $\mathbf{S}^\perp$ is $(1, 1, 1, -1)$.

c) $\mathbf{b}_1 = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{3}{2})$ $\mathbf{b}_2 = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2})$. ($\mathbf{b}_2$ is the projection of $(1, 1, 1, 1)$ onto the basis vector of $\mathbf{S}^\perp$, $\mathbf{b}_1 = (1, 1, 1, 1) - \mathbf{b}_2$.)

6. Section 5.1, Problem 3

Answer:

a) False; let $A = I$, the 2 by 2 identity.

b) True

c) False; let $A = I$, the 2 by 2 identity.

d) False; let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

7. Section 5.1, Problem 12

Answer: The correct $\det A^{-1}$ is

$$\det\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = (\frac{1}{ad-bc})^2\det\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{ad-bc}{(ad-bc)^2} = \frac{1}{ad-bc}.$$

8. Section 5.1, Problem 28

Answer:

a) True; $\det(AB)$=$\det(A)\det(B)$=0.

b) False; let $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. The product of the pivots is 1 but $\det(A) = -1$ because a row exchange was required.

c) False; let $A = 2I$ and $B = I$.

d) True; $\det(AB)$= $\det(A)\det(B)$=$\det(BA)$.

9. MATLAB

Answer: prod(diag(A))= det(original A);

sum(diag(A))= sum(diag(original A))