18.06 Spring 2006 - Problem Set 1

SOLUTIONS

1. Section 2.2, Problem 19

Answer:

a) $t(x, y, z) + (1 - t)(X, Y, Z)$ is a solution for $0 \leq t \leq 1$

b) The planes also meet on the line through the two points.

2. Section 2.2, Problem 21

Answer: After elimination, the equations are $2x + y = 0$, $\frac{3}{2}x + z = 0$, $\frac{4}{3}z + t = 0$, $\frac{5}{4}t = 5$. The pivots are $2, \frac{3}{2}, \frac{4}{3}, \frac{5}{4}$ and the solution is $t = 4$, $z = -3$, $y = 2$, $x = -1$.

3. Section 2.3, Problem 17

Answer: The linear system is:

$$a + b + c = 4$$

$$a + 2b + 4c = 8$$

$$a + 3b + 9c = 14$$

which gives $a = 2$, $b = 1$, $c = 1$.

4. Section 2.4, Problem 24

Answer:

a) $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ has $A^2 = 0$.

b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ has $A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $A^3 = 0$.

5. Section 2.5, Problem 9

Answer: Let P be the permutation matrix that swaps the first two rows of a matrix. Then $B = PA$ and $B^{-1} = A^{-1}P^{-1} = A^{-1}P = A^{-1}$ with the first two columns switched.

6. Section 2.5, Problem 28

Answer:

$$\begin{bmatrix} A & I \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 2 & 2 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 2 & 0 & -1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \to$$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} I & A^{-1} \end{bmatrix}$$

7. Section 2.5, Problem 29

Answer:

a) True; A can have at most 3 pivots

b) False; the matrix of all 1's is not invertible

c) True; $(A^{-1})^{-1} = A$

d) True; $(A^2)^{-1} = (A^{-1})^2$

8. Section 2.5, Problem 30

Answer: $c = 0$ has a column of 0's; $c = 2$ has two equal rows; $c = 7$ has two equal columns.

9. MATLAB

Answer: The mean of the square of product of the pivots approaches $n!$; so for $n = 3$, the mean approaches 6.

Code:

```
for i=1:1000
    A= randn(3);
    A(2,:)=A(2,:)-(A(2,1)/A(1,1))*A(1,:);
    A(3,:)=A(3,:)-(A(3,1)/A(1,1))*A(1,:);
    A(3,:)=A(3,:)-(A(3,2)/A(2,2))*A(2,:);
    pivots=diag(A);
    v(i)= prod(pivots)^2;
end
mean(v)
```