18.06 Professor Strang Quiz 2 April 1, 2005

Grading
1
2
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6

Your PRINTED name is: ____________________

Please circle your recitation:

1)	M 2	2-131	A. Chan	2-588	3-4110	`alicec`
2)	M 3	2-131	A. Chan	2-588	3-4110	`alicec`
3)	M 3	2-132	D. Testa	2-586	3-4102	`damiano`
4)	T 10	2-132	C.I. Kim	2-273	3-4380	`ikim`
5)	T 11	2-132	C.I. Kim	2-273	3-4380	`ikim`
6)	T 12	2-132	W.L. Gan	2-101	3-3299	`wlgan`
7)	T 1	2-131	C.I. Kim	2-273	3-4380	`ikim`
8)	T 1	2-132	W.L. Gan	2-101	3-3299	`wlgan`
9)	T 2	2-132	W.L. Gan	2-101	3-3299	`wlgan`

1 (17 pts.) If the output vectors from Gram-Schmidt are

$$q_1 = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \quad \text{and} \quad q_2 = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

describe all possible input vectors a_1 and a_2.

2 (15 pts.) If a and b are nonzero vectors in $\mathbf{R}^n$, what number x minimizes the squared length $\|b - xa\|^2$? Explain your answer.

3 (17 pts.) Find the projection p of the vector $b = (1, 2, 6)$ onto the plane $x + y + z = 0$ in $\mathbf{R}^3$. (You may want to find a basis for this 2-dimensional subspace, even an orthogonal basis.)

4 (17 pts.) Find the determinants of A and A^{-1} and the $(1,2)$ entry of A^{-1} if

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 3 \\ 1 & 3 & 1 & 9 \end{bmatrix}.$$

5 (17 pts.) Suppose the orthonormal vectors q_1 and q_2 in $\mathbf{R}^4$ are the columns of Q.

(a) Find the determinants of QQ^{T} and the 4 by 4 block matrix $[\ Q \quad Q\ ]$.

(b) For any vector b in $\mathbf{R}^4$, find the projection p of b onto the column space of Q.

(c) Which fundamental subspace contains the error vector $e = b - p$? Verify that this is true using your formula for the projection p in part (b).

6 (17 pts.) Suppose P_1 is the projection matrix onto the 1-dimensional subspace spanned by the first column of A. Suppose P_2 is the projection matrix onto the 2-dimensional column space of A. After thinking a little, compute the product P_2P_1.

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}.$$