18.06 - Spring 2005 - Problem Set 5

This problem set is due Wednesday (March 16th), at 4 PM, in 2-106. Make sure to PRINT your **name, recitation number and instructor** on your homework!

Please staple your MATLAB solutions as first pages of your homework.

Lecture 15:

- **Read:** book section 4.3.
- **Work:** book section 4.3 (exercises 4, 9, 12, 26 and 27)

Lecture 16:

- **Read:** book section 4.4.
- **Work:** book section 4.4 (exercises 3, 7, 15, 18, 24 and 36).

Lecture 17:

- **Read:** book section 5.1.
- **Work:** book section 5.1 (exercises 3, 12, 15, 28 and 34).

Challenge Problem with MATLAB

The command `a=ones(n,1)` produces an $n \times 1$ matrix of 1's.

The command `l=(1:n)'` produces the vector $(1, 2, \ldots, n)$, transposed to a column by `'`.

The command `s=l.^2` produces the vector $\left(1^2, 2^2, \ldots, n^2\right)^T$, because the dot means "a component at a time."

This problem looks for the line $y = c + dt$ closest to the parabola $y = t^2$ on the interval $t = 0$ to $t = 1$.

1. Find the best line by calculus, not MATLAB. Choose c and d to minimize
$$E(c, d) := \int_0^1 \left(c + dt - t^2\right)^2 dt$$
2. With $n = 10$, choose C and D to give the line $y = C + Dt$ that is closest to t^2 at the points $t = \frac{1}{10}, \frac{2}{10}, \ldots, 1$ (in the vector `l/10`). The unsolvable equations $AX = b$ (use least squares) are
$$[\,a \quad l/n\,] \begin{bmatrix} C \\ D \end{bmatrix} = s/n\hat{}2$$
Find the best C and D and the errors $c - C$ and $d - D$.
3. Repeat for $n = 20$. (Notice how `l/n` and `s/n^2` end at 1, like the calculus problem.)
Are the differences $c - C$ and $d - D$ smaller for $n = 20$ and by approximately what factor?