18.06 - Spring 2005 - Problem Set 3

This problem set on lectures 7 – 9 is due Wednesday (February 23th), at 4 PM, at 2-106. Make sure to include your **name and recitation number** in your homework! The numbers of the sections and exercises refer to " Introduction to Linear Algebra, **3rd Edition**, by Gilbert Strang.".

Please staple your solution as first page of your homework. Remember to PRINT your name, Recitation number and Instructor name.

Lecture 7:

- **Read:** book section 3.2.
- **Work:** book section 3.2 (exercises 9, 15, 18, 20, 23, 27, and 28).

Lecture 8:

- **Read:** book section 3.3.
- **Work:** book section 3.3 (exercises 8, 13, 17, 18, and 19).

Lecture 9:

- **Read:** book section 3.4.
- **Work:** book section 3.4 (exercises 1, 6, 10, 24 and 31).

Challenge Problem 1

Suppose R (an $m \times n$ matrix) is in row reduced echelon form $\begin{pmatrix} I & F \\ 0 & 0 \end{pmatrix}$, with r nonzero rows and first r pivot columns.

a) Describe the column space and nullspace of R.

b) Do the same for the $m \times 2n$ matrix $B = \begin{pmatrix} R & R \end{pmatrix}$.

c) Do the same for the $2m \times n$ matrix $C = \begin{pmatrix} R \\ R \end{pmatrix}$.

d) Finally, do the same for the $2m \times 2n$ matrix $D = \begin{pmatrix} R & R \\ R & R \end{pmatrix}$.

Challenge Problem 2

a) Suppose that A is a 3×3 matrix. What relation is there between the nullspace of A and the nullspace of A^2? How about the nullspace of A^3?

b) The set of polynomials of degree at most four in the variable x is a vector space. What is the nullspace of $\frac{d^2}{dx^2}$? What is the nullspace of $\left(\frac{d^2}{dx^2}\right)^2$?