18.06 Problem Set 6

Due Wednesday, May 1

Problem 1. Find a diagonalization for A by an orthogonal matrix Q where

$$A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}.$$

Problem 2. Find the singular value decomposition for the matrix

$$B = \begin{bmatrix} 1 & -1 \\ 2 & -6 \end{bmatrix}.$$

Problem 3. Suppose $L : \mathbb{R}^4 \longrightarrow \mathbb{R}^2$ is a linear transformation with $L(1,0,0,1) = (2,3)$ and $L(0,1,1,0) = (1,5)$. Find $L(6,2,2,6)$.

Problem 4. Let $L : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ by

$$L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ 2x + y \\ x + y \end{bmatrix}.$$

Let S, T be the standard bases of $\mathbb{R}^2, \mathbb{R}^3$ respectively, and $S' = \{(1,-1),(0,1)\}$, $T' = \{(1,1,0),(0,1,1),(1,-1,1)$. Find the matrix for the linear transformation L with respect to

(a) S and T.

(b) S' and T'.

(c) S and T'.

(d) S' and T.

Problem 5. Let M_2 be the (four dimensional) vector space of 2×2 matrices with real number entries. Define a linear transformation $L : M_2 \longrightarrow M_2$ by $L(A) = A^T$. Let

$$S = \{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\}$$

and

$$T = \{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\}$$

Find the matrix for L with respect to

(a) S.

(b) T.

(c) S and T.

(d) T and S.