18.06 Midterm Exam 2 (Make-up), Spring, 2001

Name ______________________ Optional Code ______________

Recitation Instructor ______________ Email Address ______________

Recitation Time ________________

This midterm is closed book and closed notes. No calculators, laptops, cell phones or other electronic devices may be used during the exam.

There are 3 problems. Good luck.

1. (30pts.)

   (a) Can $\mathbf{v} = (1, 2, 3)$ be in the nullspace and also in the column space of A? Give an example to prove yes, or a reason to prove no.

   (b) In $\mathbb{R}^4$, find the projections of $\mathbf{b} = (1, 2, 2, 7)$ onto the line through $\mathbf{a} = (1, 1, 1, 1)$ and also onto the plane $x_1 + x_2 + x_3 + x_4 = 0$.

   (c) If you can solve $A\mathbf{x} = \mathbf{b}$, then $\mathbf{b}$ must be perpendicular to ______________________.

2. (40pts.) Consider the matrix

$$A = \begin{pmatrix} 1 & -2 & 1 & -1 \\ 3 & -6 & 2 & 0 \\ 0 & 0 & 1 & -2 \end{pmatrix}$$

(a) Find the rank of A.

(b) Find a basis for the row space of A, and find a basis for the nullspace of A. What is the dimension of the nullspace of A?

(c) What can you say about the relation between the rank and the dimension of the nullspace of A?

(d) Verify that all vectors in your basis of the nullspace are orthogonal to all vectors in your basis of the row space.

3. (30pts.) We look for the line $y = C + Dt$ closest to the 3 points, $(t, y) = (0, -1)$ and $(1, 2)$ and $(2, -1)$.

   (a) If the line went through all those points (it doesn't), what three equations would need to be solved?

   (b) Find the best C and D by the least squares method.

   (c) Explain the result you get for C and D: How is the vector $\mathbf{b} = (-1, 2, -1)$ related to the plane you are projecting onto?

   (d) What is the length of the error vector $\mathbf{e}$ ($=$ distance to plane $= \|\mathbf{b} - A\mathbf{x}\|$)?