18.06 Exam 3 May 3, 2000 Closed Book

Your name is: ______________________

Please circle your recitation:

1)	M 2	2–131	P. Clifford	2)	M 3	2–131	P. Clifford
3)	T 11	2–132	T. de Piro	4)	T 12	2–132	T. de Piro
5)	T 1	2–131	T. Bohman	6)	T 1	2–132	T. Pietraho
7)	T 2	2–132	T. Pietraho	8)	T 2	2–131	T. Bohman

Note: Make sure your exam has 4 problems.

Problem		**Points possible**
1	______	25
2	______	25
3	______	25
4	______	25
Total	______	100

1 (25 pts) Let

$$A = \begin{bmatrix} 5 & 3 & 0 \\ 3 & 5 & 0 \end{bmatrix}, \quad \text{so} \quad A^{\mathrm{T}}A = \begin{bmatrix} 34 & 30 & 0 \\ 30 & 34 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The following are eigenvectors of $A^{\mathrm{T}}A$:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

(a) What are the eigenvalues of $A^{\mathrm{T}}A$?

(b) What are the singular values of A?

(c) Give the singular value decomposition of A.

Note: You must show your work to receive credit for this problem.

2 (25 pts) True (give a reason) or False (give a counterexample):

(a) If A is a symmetric matrix, any two eigenvectors of A are perpendicular.

(b) If A is $n \times n$ and has n orthonormal eigenvectors, then A is symmetric.

(c) Any eigenvector matrix S of a symmetric matrix is symmetric.

Note: You must show your work to receive credit for this problem.

3 (25 pts) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & d & 4 \\ 3 & 4 & 5 \end{bmatrix}.$$

For what (if any) values of d does A have all positive eigenvalues? (Hint: Do not try to compute the eigenvalues of A.)

Note: You must show your work to receive credit for this problem.

4 (25 pts) Suppose A is a 3×3 matrix with eigenvalues $\lambda = 1$ and $\lambda = 2$. Suppose also that $A - I$ has rank one.

(a) Which eigenvalue of A is repeated? **Explain why.**

(b) Write down a specific matrix which is similar to A and symmetric. **Explain why** they are similar.

(c) Write down a specific matrix which is similar to A and not symmetric. **Explain why** they are similar.

(d) Write down a specific matrix which has the same eigenvalues as A but is not similar to A. **Explain why** they are not similar.