18.06 Quiz 2 November 10, 1999 Closed Book

Your name is: ______________________ **Grading** 1 2 3 4

Please circle your recitation:

1)	M 2	2-131	W. Fong	2)	M 2	2-132	L. Nave
3)	M 3	2-131	W. Fong	4)	T 10	2-131	H. Matzinger
5)	T 10	2-132	P. Clifford	6)	T 11	2-131	H. Matzinger
7)	T 11	2-132	P. Clifford	8)	T 12	2-132	M. Skandera
9)	T 12	2-131	V. Kac	10)	T 1	2-131	H. Matzinger
11)	T 2	2-132	M. Skandera				

1 (25 pts.) (a) Find equations (**do not solve**) for the coefficients C, D, E in $b = C + Dt + Et^2$, the parabola which best fits the four points $(t, b) = (0, 0), (1, 1), (1, 3)$ and $(2, 2)$.

(b) In solving this problem you are projecting the vector $b =$ ______ onto the subspace spanned by ______. The projection in terms of C, D, E is $p =$ ______.

2 (28 pts.) Let

$$A = \begin{bmatrix} 3 & 4 & 6 \\ 0 & 1 & 0 \\ -1 & -2 & -2 \end{bmatrix}.$$

(a) Find the eigenvalues of the singular matrix A.

(b) Find a basis of $\mathbb{R}^3$ consisting of eigenvectors of A.

(c) By expressing $(1, 1, 1)$ as a combination of eigenvectors or by diagonalizing $A = S\Lambda S^{-1}$, compute

$$A^{99} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

3 (25 pts.) Start with two vectors (the columns of A):

$$a_1 = \begin{bmatrix} \cos\theta \\ 0 \\ \sin\theta \end{bmatrix} \quad \text{and} \quad a_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(a) With $q_1 = a_1$ find an orthonormal basis q_1, q_2 for the space spanned by a_1 and a_2 (column space of A).

(b) What shape is the matrix R in $A = QR$ and why is $R = Q^T A$? Here Q has columns q_1 and q_2. Compute the matrix R.

(c) Find the projection matrices P_A and P_Q onto the column spaces of A and Q.

4 (22 pts.) (a) If Q is an orthogonal matrix (square with orthonormal columns), show that $\det Q = 1$ or -1.

(b) How many of the 24 terms in $\det A$ are nonzero, and what is $\det A$?

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$