18.06 Quiz 1 October 8, 1999 Closed Book

Your name is: ____________________

Grading 1
2
3
4

Please circle your recitation:

1)	M 2	2-131	W. Fong	2)	M 2	2-132	L. Nave
3)	M 3	2-131	W. Fong	4)	T 10	2-131	H. Matzinger
5)	T 10	2-132	P. Clifford	6)	T 11	2-131	H. Matzinger
7)	T 11	2-132	P. Clifford	8)	T 12	2-132	M. Skandera
9)	T 12	2-131	V. Kac	10)	T 1	2-131	H. Matzinger
11)	T 2	2-132	M. Skandera				

1 (25 pts.) Suppose that row operations (elimination) reduce the matrices A and B to the same row echelon form

$$R = \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Which of the four subspaces are sure to be the same for A and B? ($C(A) = C(B)$? $N(A) = N(B)$? $C(A^T) = C(B^T)$? $N(A^T) = N(B^T)$?)

(b) Each time the subspaces in part (a) are the same for A and B, find a basis for that subspace.

(c) True or False (A is any matrix and x, y are two vectors): If Ax and Ay are linearly independent then x and y are linearly independent.

2 (25 pts.) Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 7 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 4 & 5 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(a) Find a basis for the nullspace of A.

(b) Find a basis for the column space of A.

(c) Give the complete solution to

$$Ax = \begin{bmatrix} 3 \\ 3 \\ 21 \end{bmatrix}.$$

3 (25 pts.) Suppose A is a 3 x 5 matrix and the solutions to $A^T y = 0$ are spanned by the vectors

$$y = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

(a) What is the rank of this A?

(b) For all A, why does the rank of A equal the rank of the block matrix

$$B = \begin{bmatrix} A & A \\ A & A \end{bmatrix}?$$

(c) If the rank of a matrix A equals the number of rows ($r = m$), what do we know about the equation $Ax = b$?

4 (25 pts.) Suppose A is a 4 by 3 matrix, and the complete solution to

$$Ax = \begin{bmatrix} 1 \\ 4 \\ 1 \\ 1 \end{bmatrix} \quad \text{is} \quad x = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c_1 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

(a) What is the third column of A?

(b) What is the second column of A?

(c) Give all known information about the first column of A.