6 November 1996 **Profs. S. Lee and M. Hopkins**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Hour Exam II for Course 18.06: Linear Algebra

Recitation Instructor: Your Name:

Recitation Time: Lecturer:

Grading

1.

2.

3.

4.

TOTAL:

Do all your work on these pages.
No calculators or notes.
Please work carefully, and check your intermediate results whenever possible.
Point values (total of 100) are marked on the left margin.

1. Let $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix}$.

[8] **1a.** $\det(A) =$ _______________.

[8] **1b.** $\det(\frac{1}{2}A) =$ _______________.

2. Let $A = \begin{bmatrix} 13 & 12 \\ 12 & 13 \end{bmatrix}$.

[8] **2a.** Find the eigenvalues of A.

[8] **2b.** Find the eigenvectors of A.

[12] **2c.** Find a matrix B such that $B^2 = A$.

3. Let $A = \begin{bmatrix} 1 & 4 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$.

[12] **3a.** Give a factorization $A = QR$ where R is an upper triangular matrix and Q is a matrix with orthonormal columns.

[8] **3b.** Find the least squares solution to $Ax = b$, where $b = \begin{bmatrix} 4 \\ 8 \\ 6 \end{bmatrix}$.

[16] **3c.** The projection matrix $P = A\left(A^TA\right)^{-1}A^T$ projects all vectors onto the column space of A.

Find a vector b <u>not</u> in the column space of A, and which gives $Pb = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$.

4. Suppose P is a rank one projection matrix and that the vector Px is always orthogonal to the vectors $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$.

[8] **4a.** Give an orthonormal basis for the left nullspace of P.

[12] **4b.** The projection matrix P is _____(give the 3 by 3 matrix)_____.