18.06 Problem Set 8

Due Wednesday, Nov. 15, 2006 at **4:00 p.m.** in 2-106

Problem 1 *Monday 11/6*

Do Problem #12 from section 8.3 in your book.

Problem 2 *Monday 11/6*

Of 300 million Americans, 60% own their own home and the other 40% rent.
Let's represent these proportions as a vector: $x = \begin{bmatrix} \text{owners} \\ \text{renters} \end{bmatrix} = \begin{bmatrix} .60 \\ .40 \end{bmatrix}$.
Every year, some proportion of renters will buy a house, and some proportion of homeowners will move to a rental. If these proportions remain constant, we can model this with the "Markov process" $x_{k+1} = Ax_k$ for some 2-by-2 Markov matrix A.
Suppose the proportion of homeowners and renters is modeled by this Markov process;
it maintains the steady state $x = \begin{bmatrix} .60 \\ .40 \end{bmatrix}$ above;
and 90 percent of homeowners in any given year k still own a home in year $k+1$.
Determine A, and estimate how many American renters will buy a home this year.

Problem 3 *Wednesday 11/8*

Find the first three nonzero terms in the Fourier series for the period-2π function

$$f(t) = \begin{cases} 1, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

Then find the lengths of the original function $\|f(t)\|$ and your three-term approximation $\|g(t)\|$, and the distance $\|f(t) - g(t)\|$ between them.

Problem 4 *Wednesday 11/8*

Do Problem #1 from section 10.2 in your book.

Problem 5 *Wednesday 11/8*

Do Problem #2 from section 10.2 in your book.

Problem 6 *Wednesday 11/8*

Do Problem #17 from section 10.2 in your book.

Problem 7 *Wednesday 11/8*

Do Problem #31 from section 10.2 in your book.
(Hints: U is a ____ matrix, so $U^H U =$ ____ . Λ is a ____ matrix, so $\Lambda^H \Lambda$ and $\Lambda\Lambda^H$ are ____ .)

Problem 8 *Wednesday 11/8*

Do Problem #7 from section 10.3 in your book.

Problem 9 *Monday 11/13*

Do Problem #16 from section 6.3 in your book.

Problem 10 *Monday 11/13*

Do Problem #22 from section 6.3 in your book.
Then solve $u' = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} u$ for initial condition $u(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.
Is the solution stable as $t \to \infty$? Why or why not?