18.06 Problem Set 3

Due Wednesday, Sept. 27, 2006 at **4:00 p.m.** in 2-106

Problem 1 *Monday 9/18*

Look at Worked Example 3.1B from section 3.1 in your book.
For each of the vector spaces $\mathbf{V_1}$ through $\mathbf{V_4}$,
describe a subspace, different from the examples in the text, in two different ways:
all combinations of ... = all solutions to ...

Problem 2 *Monday 9/18*

(a) Do Problem #19 from section 3.1 in your book.
(b) Also describe the nullspaces of each matrix.

Problem 3 *Wednesday 9/20*

Suppose the m-by-n matrix A ($m < n$) has a *right inverse* B, that is, a matrix B such that $AB = I$, the identity.
(a) What must the dimensions of B and of I be?
(b) Try calculating B in Matlab: let $A = \begin{bmatrix} 2 & 3 & -5 \\ 0 & -1 & 2 \end{bmatrix}$ and find `A\I`. (The identity is `eye(k)` in Matlab.)
(c) Now try calculating B another way, with `rref([A I])`. (This is the reduced-row echelon form, the result of Gauss-Jordan elimination.) What do you get? Now state another, different, B with $AB = I$. (*Hint:* Not all the rows of B are shown, unlike the square case.)
(d) Why can't there be a left inverse $CA = I$? And what would the dimensions of C and I be if there were?

Problem 4 *Wednesday 9/20*

Do Problem #23 from section 3.2 in your book.

Problem 5 *Wednesday 9/20*

Let $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Describe the nullspace of the matrix vv^{T} geometrically.

Problem 6 *Friday 9/22*

Do Problem #3 from section 3.3 in your book.

Problem 7 *Friday 9/22*

Let $A=\begin{bmatrix} -1 & 2 & 5 & 0 & 5 \\ 2 & 1 & 0 & 0 & -15 \\ 6 & -1 & -8 & -1 & -47 \\ 0 & 2 & 4 & 3 & 16 \end{bmatrix}$.[1]

(a) Reduce A to (ordinary) echelon form.
(b) What are the pivots? What are the free variables?
(c) Now reduce A to row-reduced echelon form.
(d) Give the special solutions. What is the nullspace $N(A)$?
(e) What is the rank of A?
(f) Give the complete solution to $Ax = b$, where $b = A\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Problem 8 *Friday 9/22*

Do Problem #5 from section 3.4 in your book.

Problem 9 *Friday 9/22*

Do Problem #13 from section 3.4 in your book.
(Answer in back of book, but try to do it yourself first.)

Problem 10 *Friday 9/22*

Do Problem #32 from section 3.4 in your book.

[1] Modified 12/24—thanks to Laura Garrity for pointing out the original solution's error. Hopefully it's fixed now.