18.06 Problem Set 10

Due Wednesday, Nov. 29, 2006 at **4:00 p.m.** in 2-106

Problem 1 *Monday 11/20*

Do Problem #7 from section 8.1 in your book.

Problem 2 *Wednesday, 11/22*

Do Problem #3 from section 6.6 in your book.

Problem 3 *Wednesday, 11/22*

Do Problem #11 from section 6.6 in your book.

Problem 4 *Wednesday, 11/22*

Do Problem #12 from section 6.6 in your book.

Problem 5 *Monday, 11/27*

Do Problem #7 from section 6.7 in your book.

Problem 6 *Monday, 11/27*

Do Problem #12 from section 6.7 in your book.

Problem 7 *Monday, 11/27*

Do Problem #15 from section 6.7 in your book.

Problem 8 *for Wednesday, 11/29*

One way of thinking about matrix multiplication is as a *linear transformation* — just as $y = ax$ is a linear function transforming an input x to an output y, we can think of $y = Ax$ as a "linear" function, transforming our input vector x (in R^n) to output vector y (in R^m).
Formally, a function $y = T(x)$ is "linear" if

- $T(u + v) = T(u) + T(v)$ (we can break up sums)
- $T(cv) = cT(v)$ (we can pull out constant factors)

So, for example, $f(x) = 3x$ is linear, because $3(x_1 + x_2) = 3x_1 + 3x_2$ and $3(cx) = c \cdot 3x$; but $f(x) = \sin(x)$ isn't linear, because $\sin(a + b) \neq \sin(a) + \sin(b)$.
Which of these are linear? Why or why not?

1. $y = x^2$ (input and output are in R)
2. $g(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = x_1 + 3x_2$ (input in R^2, output in R)

3. $f(x) = 3x + 1$ (careful!)

4. $T(x) = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} x$ (input and output are in R^2)

5. $L(f) = \int_0^1 f(t)\,dt$ (input is in a function space, output is in R)

6. $M(f) = \frac{d^2 f}{dt^2}$ (input and output are both functions)

Problem 9 *for Wednesday, 11/29*

If we pick a basis for the input and the output, we can write a linear transformation as a matrix. If $T(u) = \begin{bmatrix} 3 \\ -2 \\ -5 \end{bmatrix}$ and $T(v) = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, then what is $T(au + bv)$? Now write down a matrix A for which $A\begin{bmatrix} a \\ b \end{bmatrix} = T(au + bv)$.