18.06 Problem Set 1

Due Wednesday, Sept. 13, 2006 at **4:00 p.m.** in 2-106

Problem 1 *Wednesday 9/06*

Go read the Worked Examples 2.1A and 2.1B (page 29).
(You don't have to hand anything in for this problem.)

Problem 2 *Wednesday 9/06*

Write the product $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} \pi \\ e \end{bmatrix}$ in two ways:
(a) as dot products of the rows with the column vector
(b) as a linear combination of the columns.

Problem 3 *Wednesday 9/06*

(a) What matrix A takes $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ to $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$?
(b) What is $A\begin{bmatrix} 1 \\ 2 \end{bmatrix}$?

Problem 4 *Wednesday 9/06*

Do Problem #25 from section 2.1 in your book.

Problem 5 *Wednesday 9/06*

Let's practice using Matlab by multiplying a random pair of upper-triangular matrices. (Hint: you can type `diary` at the beginning of your session to save a transcript.)
We'll need two matrices. *First you pick one:* Let `A=[a b c; 0 d e; 0 0 f]`, where `a...f` are six of your favorite *nonzero* numbers. *Now let the computer pick one:* `B=rand(3,3)` gives us a random 3-by-3 matrix; we can zero out the extra coefficients one-by-one by typing e.g. `B(3,2)=0`, or all at once by keeping only the upper-triangular part `B=triu(B)`
Now compute `A*B` *and* `B*A`. What shape is this new matrix? Are AB and BA equal?

Problem 6 *Friday 9/08*

(a) Write examples of systems $A\vec{x} = \vec{b}$ where A is a 3-by-3 matrix and:

1. in the row picture, all three planes are parallel but distinct
2. all three planes are equal
3. the three planes meet in a common line
4. in the column picture, $\vec{b}$ is a linear combination of the first two columns of A.
5. $\vec{b}$ is not a linear combination of the columns of A.

(b) How many solutions for each of these? Describe the shape (point, line, ...) of each solution set.
(c) Reduce each by elimination (you need not back-substitute) and check your answer.
Circle the pivots.

Problem 7 *Friday 9/08*

Do Problem #6 from section 2.2 in your book.

Problem 8 *Friday 9/08*

Consider the system of equations

$$\begin{aligned} 2x + y + z &= -1 \\ x - z &= 0 \\ 6x + 2y + z &= -1 \end{aligned}$$

Solve this system. (Eliminate, then back-substitute.)
Circle the pivots as you find them.
Write down the elimination matrices E_{12}, E_{13}, E_{23} you used.

Problem 9 *Friday 9/08*

Do Problem #22 from section 2.2 in your book.

Problem 10 *Monday 9/11*

Consider the matrices $A=\begin{bmatrix} 0 & 1 & 4 \\ -2 & 3 & 6 \\ 2 & -1 & 2 \end{bmatrix}$, $B=\begin{bmatrix} -3 & 2 \\ 0 & 6 \\ 1 & 0 \end{bmatrix}$ and $C=\begin{bmatrix} 0 & -1 \\ 4 & 2 \\ 0 & 1 \end{bmatrix}$.
(a) Find AB and AC.
(b) What happens?
(c) Why does this tell you A is not invertible?

Problem 11 *Monday 9/11*

Do Problem #35 from section 2.4 in your book.