18.06 Professor Strang Quiz 3 December 5, 2005

Your PRINTED name is: ____________________

Grading
1
2
3

Please circle your recitation:

1)	M 2	2-131	P. Lee	2-087	2-1193	`lee`
2)	M 2	2-132	T. Lawson	4-182	8-6895	`tlawson`
4)	T 10	2-132	P-O. Persson	2-363A	3-4989	`persson`
5)	T 11	2-131	P-O. Persson	2-363A	3-4989	`persson`
6)	T 11	2-132	P. Pylyavskyy	2-333	3-7826	`pasha`
7)	T 12	2-132	T. Lawson	4-182	8-6895	`tlawson`
8)	T 12	2-131	P. Pylyavskyy	2-333	3-7826	`pasha`
9)	T 1	2-132	A. Chan	2-588	3-4110	`alicec`
10)	T 1	2-131	D. Chebikin	2-333	3-7826	`chebikin`
11)	T 2	2-132	A. Chan	2-588	3-4110	`alicec`
12)	T 3	2-132	T. Lawson	4-182	8-6895	`tlawson`

1 (30 pts.) a) Find the eigenvalues and eigenvectors of the Markov matrix

$$A = \begin{bmatrix} .9 & .4 \\ .1 & .6 \end{bmatrix}$$

b) What is the limiting value of $A^k \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ as the power k goes to infinity?

c) What does it mean to say that "*A is similar to B*"?

Is that 2 by 2 matrix A similar (yes or no) to its transpose B?

$$B = \begin{bmatrix} .9 & .1 \\ .4 & .6 \end{bmatrix}$$

Give a reason for your answer.

$\mathcal{X}$

2 (40 pts.) This 4 by 4 matrix H is a Hadamard matrix:

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Key Properties

$$H^{\mathrm{T}} = H \quad \text{and} \quad H^2 = 4I$$

a) Figure out the eigenvalues of H. Explain your reasoning.

b) Figure out H^{-1} and the determinant of H. Explain your reasoning.

c) This matrix S contains three eigenvectors of H. Find a 4th eigenvector x_4 and explain your reasoning:

$$S = \begin{bmatrix} 1 & 1 & 0 & \\ 1 & 0 & -1 & \\ 1 & 0 & 1 & \\ -1 & 1 & 0 & \end{bmatrix}$$

d) Find the solution to $du/dt = Hu$ given that $u(0) =$ third column of S.

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3 (30 pts.) Suppose A is a 3 by 3 symmetric matrix with eigenvalues $2, 5, 7$ and corresponding eigenvectors x_1, x_2, x_3.

a) Suppose x is a combination $x = c_1x_1 + c_2x_2 + c_3x_3$. *Find* Ax. Now find $x^{\mathrm{T}}Ax$ using the symmetry of A. Prove that $x^{\mathrm{T}}Ax > 0$ (unless $x = 0$).

b) Suppose those eigenvectors have length 1 (unit vectors). Show that $B = 2x_1x_1^{\mathrm{T}} + 5x_2x_2^{\mathrm{T}} + 7x_3x_3^{\mathrm{T}}$ has the same eigenvectors and eigenvalues as A. Is B necessarily the same matrix as A (yes or no)?

c) For which numbers b does this matrix have 3 positive eigenvalues?

$$A = \begin{bmatrix} 2 & b & 3 \\ b & 2 & b \\ 3 & b & 4 \end{bmatrix}$$

Note: The SVD will be on the final when you have more time to digest it.

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