18.06 Problem Set 8

Due at 4pm on Wednesday, November 16 in 2-106

Please PRINT your name and recitation information on your homework

1. Section 6.2, Problem 42
2. Section 6.3, Problem 3
3. Section 6.3, Problem 10
4. Section 6.3, Problem 11
5. Section 6.3, Problem 23
6. Section 8.3, Problem 7
7. Section 8.3, Problem 8
8. Section 8.3, Problem 9
9. Section 8.3, Problem 10
10. Section 8.3, Problem 12
11. Define the matrix A_n as follows:

$$A_n = \frac{1}{n+1}\begin{bmatrix} \frac{(n+1)(n+2)}{2}-1 & n+1 & n+1 & \dots & n+1 & n+1 & -\frac{n(n+1)}{2}+1 \\ \frac{(n-1)n}{2}+1 & 2(n+1) & n+1 & \dots & n+1 & n+1 & -\frac{n(n+1)}{2}+2 \\ \frac{(n-1)n}{2}+2 & -n-1 & 3(n+1) & \dots & n+1 & n+1 & -\frac{n(n+1)}{2}+3 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{n(n+1)}{2}-3 & -n-1 & -n-1 & \dots & (n-2)(n+1) & n+1 & -\frac{(n-1)n}{2}-2 \\ \frac{n(n+1)}{2}-2 & -n-1 & -n-1 & \dots & -n-1 & (n-1)(n+1) & -\frac{(n-1)n}{2}-1 \\ \frac{n(n+1)}{2}-1 & -n-1 & -n-1 & \dots & -n-1 & -n-1 & \frac{n(n+1)}{2}-1 \end{bmatrix}$$

(the numbers in the first column below the $(1,1)$ entry form an increasing arithmetic progression, and so do the numbers in the last column above the (n,n) entry). For example, here are A_4, A_5, and A_6:

$$A_4 = \frac{1}{5}\begin{bmatrix} 16 & 5 & 5 & -9 \\ 7 & 10 & 5 & -8 \\ 8 & -5 & 15 & -7 \\ 9 & -5 & -5 & 9 \end{bmatrix}; \quad A_5 = \frac{1}{6}\begin{bmatrix} 22 & 6 & 6 & 6 & -14 \\ 11 & 12 & 6 & 6 & -13 \\ 12 & -6 & 18 & 6 & -12 \\ 13 & -6 & -6 & 24 & -11 \\ 14 & -6 & -6 & -6 & 14 \end{bmatrix};$$

$$A_6 = \frac{1}{7}\begin{bmatrix} 29 & 7 & 7 & 7 & 7 & -20 \\ 16 & 14 & 7 & 7 & 7 & -19 \\ 17 & -7 & 21 & 7 & 7 & -18 \\ 18 & -7 & -7 & 28 & 7 & -17 \\ 19 & -7 & -7 & -7 & 35 & -16 \\ 20 & -7 & -7 & -7 & -7 & 20 \end{bmatrix}.$$

(a) Using MATLAB, diagonalize matrices A_4, A_5, and A_6. Recall that the command `[V,L] = eig(A)` gives the eigenvectors and the eigenvalues of A. Can you multiply each resulting eigenvector by a scalar and reorder the eigenvalues so that the answer looks nice?

(b) Predict the eigenvalues and the corresponding eigenvectors of A_n and then prove your answer.