18.06 Problem Set 5

Due at 4pm on Wednesday, October 19 in 2-106

Please PRINT your name and recitation information on your homework

1. Section 4.2, Problem 5
2. Section 4.2, Problem 13
3. Section 4.2, Problem 17
4. Section 4.2, Problem 19
5. Section 4.2, Problem 27
6. Section 4.3, Problem 1
7. Section 4.3, Problem 17
8. Section 4.3, Problem 22
9. Section 4.3, Problem 26
10. Section 4.3, Problem 27
11. Section 4.4, Problem 6
12. Section 4.4, Problem 7
13. The MATLAB command `a=ones(n,1)` produces an n by 1 matrix of 1's. The command `r=(1:n)'` produces the vector $(1, 2, \ldots, n)$, transposed to a column by `'`. The command `s=r.^2` produces the vector $(1^2, 2^2, \ldots, n^2)$, because the dot means "a component at a time."

This problem looks for the line $y = c + dt$ closest to the parabola $y = t^2$ on the interval $t = 0$ to $t = 1$.

(a) Find the best line by calculus, not MATLAB. Choose c and d to minimize

$$E(c, d) := \int_0^1 (c + dt - t^2)^2 dt$$

(b) With $n = 10$, choose C and D to give the line $y = C + Dt$ that is closest to t^2 at the points $t = \frac{1}{10}, \frac{2}{10}, \ldots, 1$ (in the vector `r/10`). The unsolvable equations $AX = b$ (use least squares) are

$$\begin{bmatrix} a & r/n \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = s/n^2$$

Find the best C and D and the errors $c - C$ and $d - D$.

(c) Repeat for $n = 20$. (Notice how `r/n` and `s/n^2` end at 1, like the calculus problem.) Are the differences $c - C$ and $d - D$ smaller for $n = 20$ and by approximately what factor?