18.06 Problem Set 4

Due at 4pm on Wednesday, October 12 in 2-106

Please PRINT your name and recitation information on your homework

1. Section 3.5, Problem 42

2. Section 3.6, Problem 17

3. Section 3.6, Problem 23

4. Section 3.6, Problem 25

5. Section 4.1, Problem 22

6. Section 4.1, Problem 26

7. Two matrices A and B have the same shape and the same nullspace.

(a) Show that A and B have the same row space.

(b) Show that A and B have the same row reduced echelon form R. (Hint: what is special about the basis for the row space given by the non-zero rows of R?)

8. Let A_n be the n by n matrix containing 1's on and above the main diagonal, and the alternating pattern of 1's and -1's below the main diagonal. As an example, A_5 and A_6 are as follows:

$$A_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 & 1 \end{bmatrix}$$

Use MATLAB to do the following:

(a) Find the LDU factorization of A_5 and A_6. (Use the `lu` command to get the standard LU factorization, then factor out the pivots.)

(b) Find A_5^{-1} and A_6^{-1}. Predict what A_n^{-1} is for a general n.

(c) Find L^{-1}, where L is as in part (a).