18.06, Fall 2004, Problem Set 8

Due before 4PM on Wednesday November 10th, 2004, in the boxes in 2-106. No late homework will be accepted. Don't forget to write your name, recitation section **and the names of students you have collaborated with** on the problem set. There is one box for each recitation section. For full credit, please be sure to show and explain your work. Exercises refer to the 3rd edition of the textbook.

Reading assignment: Sections 10.1, 6.1 and 6.2.

1. Let
$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 4 & 1 & 0 \end{bmatrix}.$$
   (a) Compute the matrix C of cofactors.
   (b) Compute $\det(A)$.
   (c) Compute A^{-1} using the cofactor formula. Verify your answer by making sure $AA^{-1} = I$.
   (d) What is $\det(-3A^4)$?

2. Using Cramer's rule, solve for just x_3 in
$$\begin{cases} 2x_1 & +x_2 & -x_3 & +x_4 & = 5 \\ -x_1 & +2x_2 & +x_3 & & = -6 \\ & -3x_2 & +x_3 & -x_4 & = 1 \\ x_1 & +7x_2 & & +3x_4 & = -1 \end{cases}$$
   Show your work.

3. Assuming we do not perform any row permutations, what is the 5th and last pivot when transforming the following matrix A into an upper triangular matrix by row eliminations? Explain.
$$\begin{bmatrix} 1 & 2 & 3 & 4 & 0 \\ 0 & 2 & 4 & 6 & 0 \\ -1 & -2 & 2 & 4 & 0 \\ 5 & 4 & 3 & 1 & 0 \\ 2 & 1 & 1 & 1 & 2 \end{bmatrix}.$$

4. Find all values, including complex values, for x and y such that
$$\begin{bmatrix} x & y & 8 \\ 1 & x & y \end{bmatrix}$$
   has rank 1.

5. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$. Compute the eigenvalues and eigenvectors of A. Show your work.

6. Let

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}.$$

(a) What is the characteristic polynomial for A (i.e. compute $\det(A - \lambda I)$)?

(b) Verify that 1 is an eigenvalue of A. What is a corresponding eigenvector?

(c) What are the other eigenvalues of A?

7. (a) Argue that the eigenvalues of a permutation matrix must all have a modulus equal to 1.

(b) What are the eigenvalues of:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}?$$

Express them in polar form.