18.06, Fall 2004, Problem Set 6

Due before 4PM on Wednesday October 27th, 2004, in the boxes in 2-106. No late homework will be accepted. Don't forget to write your name, recitation section **and the names of students you have collaborated with** on the problem set. There is one box for each recitation section. For full credit, please be sure to show and explain your work. Exercises refer to the 3rd edition of the textbook.

Reading assignment: Sections 4.3, 4.4, 7.1, 7.2.

1. Consider in R^4 the subspace given by $F = \{(x, y, z, w) : x + 2y - z + w = 0\}$.
   (a) Give a basis of F. What is the dimension of F?
   (b) Use Gram-Schmidt to transform your basis into an orthonormal basis.
   (c) What is the distance between the point $(3, 1, 1, 1)$ and (the closest point on) F?

2. From Kepler's first law, we know that the orbit of a comet should be of the form (in polar coordinates (r, θ)):
$$r = \beta + e(r \cos\theta)$$
where β is a constant and e is the eccentricity of the orbit. The eccentricity e will satisfy $0 \leq e < 1$ for elliptic orbits, $e = 1$ for parabolic orbits and $e > 1$ for hyperbolic orbits.

   Suppose that observations of a newly discovered comet gives the following data (β is in radians):

θ	0.88	1.10	1.42	1.77	2.14
r	3.0	2.3	1.65	1.25	1.01

   Using a least-squares fit, determine the (best approximation to the) eccentricity and the β value. (You are welcome to use a calculator or MATLAB, but show your computations.)

3. Consider
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$
   (a) Perform Gram-Schmidt on the columns of A. (Do not use MATLAB here, and just keep the square roots around.)
   (b) Write a QR factorization of A.

4. Exercise 14 on page 369. In this exercise the linear transformation T maps a 2×2 matrix M into the 2×2 matrix AM. V denotes the set of all 2×2 matrices.

5. Exercise 14 on page 381.