18.06, Fall 2004, Problem Set 3

Due before 4PM on Wednesday September 29th, 2004, in the boxes in 2-106. No late homework will be accepted. Don't forget to write your name, recitation section **and the names of students you have collaborated with** on the problem set. There is one box for each recitation section. For full credit, please be sure to show and explain your work. Exercises refer to the 3rd edition of the textbook.

Reading assignment: Sections 3.1-3.3.

1. Justify your answer for each question below.

   (a) Is $F = \{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x \leq y \leq z\}$ a subspace of R^3?

   (b) Is the set of all functions f of the form $f(x) = ax^2 + x$ (where a can take any value) a subspace of the vector space of all functions?

   (c) Let M be a given 3×4 matrix. Let F be the set of all 4×2 matrices such that $MF = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. Is F a subspace of all 4×2 matrices?

2. Consider the set of rotations of the plane (around the origin). For example, the rotation of $\pi/2$ sends the point $(1, 0)$ to the point $(0, 1)$, and the rotation of $\pi/4$ sends $(1, 0)$ to $(1/\sqrt{2}, 1/\sqrt{2})$. The set of rotations can be considered a vector space, if we define addition and scalar multiplication appropriately.

   (a) If x is the rotation by θ_1 radians and y is the rotation by θ_2 radians, how would you define $x + y$?

   (b) How would you define $-x$? More generally, how would you define cx where c is a scalar?

3. Suppose $A = PQ$ where A is $m \times n$, P is $m \times p$ and Q is $p \times n$. Show that $C(A) \subseteq C(P)$. Can $C(A)$ be different from $C(P)$? (Either argue why they are equal or give an example where they differ.)

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 & 0 \\ 2 & 4 & -3 & 7 & 0 \\ 3 & 6 & -5 & 10 & -2 \\ 5 & 10 & -9 & 16 & 0 \end{bmatrix}.$$

   (a) Reduce A to the ordinary echelon form.

   (b) What are the pivot variables and what are the free variables?

   (c) Further reduce the matrix into row reduced echelon form.

   (d) Give the special solutions in the nullspace of A.

   (e) What is the rank of A?

(f) Exhibit an $r \times r$ submatrix of A which is invertible, where $r = rank(A)$. (An $r \times r$ submatrix is obtained by keeping r rows and r columns of A.)

5. (a) This exercise uses MATLAB. Please provide the output of your MATLAB computations (an easy way to do this is to use the command `diary` at the beginning of your session, which saves everything in a file called *diary*). Take your 9-digit MIT ID $a_1 a_2 a_3 \cdots a_9$, where a_i is the ith digit. Construct the 9×9 matrix A with $A_{ij} = a_i a_j$, and let $B = A + A^2 + A^3$. Compute the rank of B.

   (b) Can you explain this answer?