18.06, Fall 2004, Problem Set 1

Due before 4PM on Wednesday September 15th, 2004, in the boxes in 2-106. No late homework will be accepted. Don't forget to write your name and recitation section on the problem set. There is one box for each recitation section. For full credit, please be sure to show and explain your work. Exercises refer to the 3rd edition of the textbook.

Reading assignment: Sections 1-2.4.

1. Exercise 8 from Section 1.2 on page 18.
2. Exercise 6 from Section 2.4 on Page 66.
3. Exercise 24 from Section 2.4 on page 68.
4. Solve the following system of equations by elimination and back substitution:

$$\begin{cases} x & +2y & +3z & = & 0 \\ -x & +y & -2z & = & -3 \\ 2x & +y & +z & = & 3 \end{cases}$$

   Let $Ax = b$ denote the above system. Write the three elimination matrices E_{21}, E_{31} and E_{32} that put A into triangular form U with $E_{32}E_{31}E_{21}A = U$.

   Compute $M = E_{32}E_{31}E_{21}$.

5. Consider the 3 elimination matrices of the previous exercise. Which of the products $E_{21}E_{31}$, $E_{21}E_{32}$ and $E_{31}E_{32}$ commute[1] and which do not? Explain. You actually do not need to compute the products to answer this question, you can simply reason about what elimination matrices do.

6. This exercise is to familiarize you with MATLAB on athena. First read the tutorial available on the course homepage. Now, take your 9-digit MIT ID, say $abcdefghi$, and construct the following matrix:

$$B = \frac{1}{a+b+c+d+e+f+g+h+i} \begin{bmatrix} a+b+c & e+g+h & d+f+i \\ d+e+f & a+c+i & b+g+h \\ g+h+i & b+d+f & a+c+e \end{bmatrix}.$$

   Compute, using MATLAB, B^{40} for your MIT ID (say with just 4 decimals as the usual matlab output; you could get more decimals with the `format` command) and attach the MATLAB output.

   As an example, if your MIT ID was 987654321, you could get B in MATLAB with the commands:

```
>> a=9; b=8; c=7; d=6; e=5; f=4; g=3; h=2; i=1;
>> A=[a+b+c e+g+h d+f+i; d+e+f a+c+i b+g+h; g+h+i b+d+f a+c+e]
```

[1] The product AB commutes (or the matrices A and B commute) if $AB = BA$.

```
A =

    24    10    11
    15    17    13
     6    18    21

>> B=A/(a+b+c+d+e+f+g+h+i)

B =

    0.5333    0.2222    0.2444
    0.3333    0.3778    0.2889
    0.1333    0.4000    0.4667
```

Then you would still need to compute B^{40}.