18.06 Professor Strang/Ingerman Quiz 3 December 6, 2002

Your name is: ______________________________

Please circle your recitation:

1)	M2	2-131	P.-O. Persson	2-088	2-1194	`persson`
2)	M2	2-132	I. Pavlovsky	2-487	3-4083	`igorvp`
3)	M3	2-131	I. Pavlovsky	2-487	3-4083	`igorvp`
4)	T10	2-132	W. Luo	2-492	3-4093	`luowei`
5)	T10	2-131	C. Boulet	2-333	3-7826	`cilanne`
6)	T11	2-131	C. Boulet	2-333	3-7826	`cilanne`
7)	T11	2-132	X. Wang	2-244	8-8164	`xwang`
8)	T12	2-132	P. Clifford	2-489	3-4086	`peter`
9)	T1	2-132	X. Wang	2-244	8-8164	`xwang`
10)	T1	2-131	P. Clifford	2-489	3-4086	`peter`
11)	T2	2-132	X. Wang	2-244	8-8164	`xwang`

1 (36 pts.) (a) What are the eigenvalues of the 5 by 5 matrix $A = \boldsymbol{ones}(5)$ with all entries $a_{ij} = 1$? Please look at A, not at $\det(A - \lambda I)$.

(b) Solve this differential equation to find $\boldsymbol{u}(t)$:

$$\frac{d\boldsymbol{u}}{dt} = A\boldsymbol{u} \qquad \text{starting from } \ \boldsymbol{u}(0) = (0,1,1,1,2)\,.$$

First split $\boldsymbol{u}(0)$ into two eigenvectors of A.

(c) Using part (a), what are the *eigenvalues* and *trace* and *determinant* of the matrix $B =$ same as A except zeros on the diagonal.

2 (20 pts.) (a) If A is similar to B show that e^A is similar to e^B. *First define "similar" and* e^A!!

(b) If A has 3 eigenvalues $\lambda = 0, 2, 4$, *find the eigenvalues of* e^A.

Using part (a) explain this connection with determinants:

$$\text{determinant of } e^A = e^{\text{trace of } A}$$

3 (22 pts.) Suppose the SVD $A = U\Sigma V^{\mathrm{T}}$ is

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

(a) For which angles θ and α (0 to $\frac{\pi}{2}$) is A a positive definite symmetric matrix? No computing needed.

(b) What are the eigenvalues and eigenvectors of $A^{\mathrm{T}}A$? No computing!

4 (22 pts.) Multinational companies in the US, Asia, and Europe have assets of \$ 12 trillion. At the start, \$ 6 trillion are in the US, \$ 6 trillion in Europe. Each year half the US money stays home, $\frac{1}{4}$ each goes to Asia and Europe. For Asia and Europe, half stays home and half is sent to the US.

$$\begin{bmatrix} \text{US} \\ \text{Asia} \\ \text{Europe} \end{bmatrix}_{\text{year } k+1} = \begin{bmatrix} .5 & .5 & .5 \\ .25 & .5 & 0 \\ .25 & 0 & .5 \end{bmatrix} \begin{bmatrix} \text{US} \\ \text{Asia} \\ \text{Europe} \end{bmatrix}_{\text{year } k}$$

(a) The eigenvalues and eigenvectors of this *singular* matrix A are

(b) The limiting distribution of the \$ 12 trillion as the world ends is

$$\begin{aligned} \text{US} &= \\ \text{Asia} &= \\ \text{Europe} &= \end{aligned}$$