18.06 Professor Strang/Ingerman Quiz 2 October 25, 2002

Your name is: ______________________________

Please circle your recitation:

1)	M2	2-131	P.-O. Persson	2-088	2-1194	persson
2)	M2	2-132	I. Pavlovsky	2-487	3-4083	igorvp
3)	M3	2-131	I. Pavlovsky	2-487	3-4083	igorvp
4)	T10	2-132	W. Luo	2-492	3-4093	luowei
5)	T10	2-131	C. Boulet	2-333	3-7826	cilanne
6)	T11	2-131	C. Boulet	2-333	3-7826	cilanne
7)	T11	2-132	X. Wang	2-244	8-8164	xwang
8)	T12	2-132	P. Clifford	2-489	3-4086	peter
9)	T1	2-132	X. Wang	2-244	8-8164	xwang
10)	T1	2-131	P. Clifford	2-489	3-4086	peter
11)	T2	2-132	X. Wang	2-244	8-8164	xwang

1 (40 pts.) (a) Find the projection matrix P_C onto the column space of A (after looking closely at the matrix!)

$$A = \begin{bmatrix} 3 & 3 & 6 \\ 1 & 1 & 2 \end{bmatrix}$$

(b) Find the 3 by 3 projection matrix P_R onto the row space of A. What is the closest vector in the row space to the vector $\boldsymbol{b} = (1, 0, 0)$?

(c) Multiply $P_C A$ and then $P_C A P_R$. Your answers should be a little surprising—can you explain?

(d) Find a basis for the subspace of all vectors orthogonal to the row space of A.

2 (30 pts.) (a) Choose c and the last column of Q so that you have an orthogonal matrix:

$$Q = c \begin{bmatrix} 1 & -1 & -1 & \mathrm{x} \\ -1 & 1 & -1 & \mathrm{x} \\ -1 & -1 & -1 & \mathrm{x} \\ -1 & -1 & 1 & \mathrm{x} \end{bmatrix}$$

(b) Project $\boldsymbol{b} = (1, 1, 1, 1)$ onto the first column of Q. Then project $\boldsymbol{b}$ onto the plane spanned by the first two columns.

(c) Suppose the last column of the 4 by 4 matrix (where the x's are) was changed to $(1, 1, 1, 1)$. Call this new matrix A. If Gram-Schmidt is applied to the 4 columns of A, what would be the 4 outputs $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, $\boldsymbol{q}_3$, $\boldsymbol{q}_4$? (Don't do a lot of calculations... please.)

3 (30 pts.) (a) If you multiply all $n!$ permutations together into a single P, is the product odd or even? (Answer might depend on n.)

(b) If you know that $\det A = 6$, what is the determinant of B?

$$\det A = \begin{vmatrix} \text{row } 1 \\ \text{row } 2 \\ \text{row } 3 \end{vmatrix} = 6 \qquad\qquad \det B = \begin{vmatrix} \text{row } 3 + \text{row } 2 + \text{row } 1 \\ \text{row } 2 + \text{row } 1 \\ \text{row } 1 \end{vmatrix} = ?$$

(c) Prove $\det A = 0$ for the 5 by 5 *all-ones matrix* (all $a_{ij} = 1$) in ***two ways***:

(1) Using Properties 1–10 of determinants

(2) Using the "big formula" = sum of 120 terms.