Course 18.06, Fall 2002: Quiz 1, Solutions

1 (a) For example

$$\boldsymbol{w} = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}, \boldsymbol{z} = \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix} \qquad \text{or} \qquad \boldsymbol{w} = \boldsymbol{u} + \boldsymbol{v} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}, \boldsymbol{z} = 3\boldsymbol{u} - \boldsymbol{v} = \begin{bmatrix} 5 \\ 0 \\ 6 \end{bmatrix}$$

(b), (c) For example

$$M = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 0 \\ 0 & \frac{5}{2} & 0 \\ 0 & -1 & 0 \end{bmatrix} \to \begin{bmatrix} 2 & 1 & 0 \\ 0 & \frac{5}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

x_3 free variable. Let $x_3 = 1$ then $x_1 = x_2 = 0$. Nullspace is all vectors

$$\lambda \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2 (a)

$$A = \begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 8 & 5 & 2 \\ 1 & 5 & 3 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = U$$

Pivot columns 1 and 2, free columns 3 and 4.

(b)

$$N(A) = \text{linear combination of } \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(c)

$$\text{Particular } x_p = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \text{Complete } x = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

3 (a)

$$A = \begin{bmatrix} 1 & & & \\ 1 & 1 & & \\ 1 & 2 & 1 & \\ 1 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ & 1 & 2 & 3 \\ & & 1 & 3 \\ & & & 1 \end{bmatrix}$$

(b) U has 4 nonzero entries on the diagonal
$\Rightarrow A$ has 4 nonzero pivots
$\Rightarrow$ Gauss-Jordan will work
$\Rightarrow A^{-1}$ exists

(c) If the last diagonal entry of U was zero $\Rightarrow A_{44} = 1 + 9 + 9 = 19$.

(d)

$$P = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{bmatrix} \Rightarrow PA \text{ has reversed rows \& } AP \text{ has reversed columns.}$$