18.06 Strang, Edelman, Huhtanen Final December 20, 2001

Your name is: ______________________________

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Recitations

#	Time	Room	Instructor	Office	Phone	Email `@math`
Lect. 1	MWF 12	4-270	M Huhtanen	2-335	3-7905	`huhtanen`
Lect. 2	MWF 1	4-370	A Edelman	2-380	3-7770	`edelman`
Rec. 1	M 2	2-131	D. Sheppard	2-342	3-7578	`sheppard`
2	M 2	2-132	M. Huhtanen	2-335	3-7905	`huhtanen`
3	M 3	2-131	D. Sheppard	2-342	3-7578	`sheppard`
4	T 10	2-132	A. Lachowska	2-180	3-4350	`anechka`
5	T 10	2-131	S. Kleiman	2-278	3-4996	`kleiman`
6	T 11	2-131	M. Honsen	2-490	3-4094	`honsen`
7	T 11	2-132	A. Lachowska	2-180	3-4350	`anechka`
8	T 12	2-131	M. Honsen	2-490	3-4094	`honsen`
9	T 1	2-132	A. Lachowska	2-180	3-4350	`anechka`
10	T 1	2-131	S. Kleiman	2-278	3-4996	`kleiman`
11	T 2	2-132	F. Latour	2-090	3-6293	`flatour`

For full credit, carefully explain your reasoning, as always!

1 (36 pts.) Let

$$a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \; b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \; c = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \text{ and } d = \begin{bmatrix} -7 \\ 2 \\ 2 \end{bmatrix}.$$

(a) Give d as a linear combination of a, b and c.

(b) By using Gram-Schmidt, orthogonalize a, b and c to get ortohonormal vectors q_1, q_2 and q_3.

(c) Give the change of basis matrix from the basis a, b and c to the basis q_1, q_2 and q_3.

2 (36 pts.) Let A be the 3-by-4 matrix defined by

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 + x_3 + x_4 \\ x_1 + 2x_2 - x_4 \\ x_1 + x_2 + 3x_3 - 3x_4 \end{bmatrix}.$$

(a) Give A explicitly.

(b) Find the nullspace matrix of A^T.

(c) What is the dimension of the column space of A?

3 (36 pts.) (a) Let

$$A = \begin{bmatrix} 4 & 2 \\ 0 & 4 \end{bmatrix}.$$

Find all diagonal matrices D for which DAD^{-1} is a Jordan matrix.

(b) Find all pairs $a, b \in \mathbf{R}$ such that

$$A = \begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ a & b \end{bmatrix}$$

is an orthogonal matrix.

(c) Find all orthogonal upper-triangular matrices of size 3-by-3.

4 (32 pts.) (a) Diagonalize the 3-by-3 symmetric matrix A that corresponds to the quadratic form

$$f(x, y, z) = 4xy + 3y^2 + 4z^2.$$

(In your diagonalization $A = S\Lambda S^{-1}$, choose S to be orthogonal.)

(b) Replace the $(1, 1)$-entry of A with $a > 0$. For which values of a do you get a positive definite matrix?

5 (32 pts.) Let

$$A = QR = \begin{bmatrix} 1/5 & -2/5 & -4/5 \\ 2/5 & 1/5 & 2/5 \\ 2/5 & -4/5 & 2/5 \\ 4/5 & 2/5 & -1/5 \end{bmatrix} \begin{bmatrix} 5 & -2 & 1 \\ 0 & 4 & -1 \\ 0 & 0 & a \end{bmatrix}$$

(Q has orthonormal columns).

(a) Give an orthonormal basis of $C(A)$.

(b) For which values of a the rank of A is 2?

(c) Solve $A^T x = \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix}$.

(d) Let $a = 2$. Solve $Ay = b$ in the least squares sense for

$$b = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -2 \end{bmatrix}.$$

6 (36 pts.) Let A be a symmetric matrix of size 3-by-3 with the 3rd column $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

(a) Assume A has the cofactors $C_{1,3} = 0$ and $C_{23} = 1$ and

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}.$$

Give A explicitely. Is A invertible?

(b) Compute the SVD of A.

(c) With the help of the SVD, give an orthonormal basis of the column space of A.