18.06 Professor Strang Quiz 2 October 30, 2000

Your name is: ______________________________

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12)	T2	2-132	Ingerman	2-372	3-4344	ingerman@math
13)	T2	1-150	Nave	2-251	3-4097	nave@math

1 (36 pts.) Suppose Q is a 4 by 3 matrix with orthonormal columns q_1, q_2, q_3.

(a) Starting from a vector v (not in the column space of Q), give a formula for the fourth orthonormal vector q_4 that is produced by Gram-Schmidt from q_1, q_2, q_3, v.

(b) Describe the nullspace of Q (the same 4 by 3 matrix) and the nullspace of Q^{T}. (You can answer even if you didn't find the particular formula for q_4 in part a.) Describe also the nullspaces of $Q^{\mathrm{T}}Q$ and QQ^{T}

(c) Suppose $b = q_1 + 2q_2 + 3q_3 + 4q_4$. Find the least squares solution $\bar{x}$ to $Qx = b$. What is the projection p of this b onto the column space of Q?

2 (24 pts.) (a) Fitting the best (least squares) straight line through the points $(t, b) = (2, 3), (3, 5)$, and $(4, K)$ is the same as solving what system of equations $Ax = b$ by least squares? Is there any value of K for which this system $Ax = b$ has an exact solution?

(b) For general A and B, under what condition does the equation $Ax = b$ have $\bar{x} = 0$ as its least squares solution? In the example of part (a), prove that there is or there isn't a value of K so that $\bar{x} = 0$ is the least squares solution.

3 (40 pts.) (a) Suppose A is a 4 by 4 matrix. If you add 1 to the entry a_{14} in the northeast corner, how much will the determinant change?

(b) Explain why the determinant of every projection matrix is either 0 or 1.

(c) Find the determinant of the "circulant matrix"

$$A = \begin{bmatrix} 0 & b & 0 & a \\ a & 0 & b & 0 \\ 0 & a & 0 & b \\ b & 0 & a & 0 \end{bmatrix}.$$