Circle Your Recitation:

Your Name:

Optional Code:

Grading:
1.
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18.06 Hour Exam III

22 November, 1993

Do all your work on these 5 pages. No calculators or notes. 25 points per question.

1 (a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -3 & 5 \\ 3 & -5 \end{bmatrix}$.

(b) Diagonalize A in the form $S^{-1}\Lambda S$. You do not have to calculate S^{-1} explicitly.

(c) Write down two linearly independent vector solutions $\mathbf{u}(t)$ of the equation

$$\frac{d\mathbf{u}(t)}{dt} = A\mathbf{u}(t).$$

(d) As $t \to \infty$, what is the limiting behavior of the solution to $\dfrac{d\mathbf{u}}{dt} = A\mathbf{u}$ with initial condition $\mathbf{u}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$?

2. Suppose A is a square matrix with eigenvalues $\lambda_1 = 0$, $\lambda_2 = c$ (real) and $\lambda_3 = 2$, and eigenvectors

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}.$$

In each of the following questions, you must give a reason in order to get full credit.

(a) For which values of c (if any) is A a diagonalizable matrix? Why?

(b) For which values of c (if any) is A a symmetric matrix? Why?

(c) For which values of c (if any) is A a positive definite matrix? Why?

(d) For which values of c (if any) is A a Markov matrix? Why?

(e) For which values of c (if any) is $P = \frac{1}{2}A$ a projection matrix? Why?

3. Suppose that A has eigenvalues $\lambda = 0, 1, 2$, with respective eigenvectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

(a) Describe the null space, column space, and row space of A in terms of $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

(b) Find all solutions to $A\mathbf{x} = \mathbf{v} - \mathbf{w}$.

(c) Prove that A is not an orthogonal matrix.

4. (a) The sequence of numbers $c_0, c_1, c_2, \ldots$ satisfy the recurrence relation

$$c_{n+2} = 2c_{n+1} + 3c_n.$$

Find a matrix A such that $\begin{bmatrix} c_{n+2} \\ c_{n+1} \end{bmatrix} = A \begin{bmatrix} c_{n+1} \\ c_n \end{bmatrix}$.

(b) Find the eigenvalues and eigenvectors of A.

(c) If $c_0 = 2$, find a value of c_1 such that the sequence of numbers c_n does not blow up (i.e. $|c_n| \leq$ constant).