18.06 Quiz 2 Professor Strang

1. (18pts) (a) If an m by n matrix Q has orthonormal columns is the matrix Q necessarily invertible? Give a reason or a counterexample.

   (b) What is the nullspace of Q (and WHY)?

   (c) What is the projection matrix onto the column space of Q? Avoid inverses where possible.

2. (30pts) We look for the line $y = C + Dt$ closest to 3 points $(t, y) = (0, -1)$ and $(1, 2)$ and $(2, -1)$.

   (a) If the line went through those points (it doesn't), what three equations would be solved?

   (b) Find the best C and D by the least squares method.

   (c) Explain the result you get for C and D: How is the vector $b = (-1, 2, -1)$ related to the plane you are projecting onto?

   (d) What is the length of the error vector e (= distance to plane = $\|b - A\bar{x}\|$).

3. (22pts) The problem is to find the determinants of

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} x & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

   (a) Find $\det A$ and give a reason.

   (b) Find $\det B$ using elimination.

   (c) Find $\det C$ for any value of x. For this you could use Property 1 of the determinant.

4. (30pts) (a) Decide if A is singular or invertible. $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ 2 & 2 & -4 \end{bmatrix}$.

   (b) Find an orthonormal basis for its column space (if such a basis exists).

   (c) Why does $P = A(A^TA)^{-1}A^T$ *not* give the projection matrix onto the column space of A?

   Find that projection matrix somehow.