18.06 Hour Test 1 in Linear Algebra: MIT Course 18.06 October 13, 1993

(1a) Elimination matrices E_{21} and E_{32} will reduce A to triangular form. Find M so that $MA = U$ is upper triangular, if A is

$$\begin{bmatrix} 2 & 1 & 1 \\ 2 & 4 & 3 \\ 0 & 9 & 9 \end{bmatrix}.$$

(1b) Find a matrix L so that $A = LU$.

2.

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 5 & 4 \\ 3 & 3 & 9 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} 3 \\ 6 \\ \mathrm{c} \end{bmatrix}$$

(2a) Find a basis for the nullspace of A.

(2b) For which number c is the vector b in the column space of A?

(2c) Find the complete (general) solution to $Ax = b$ when c is chosen so that this equation is solvable.

(3a) Find a basis for the space of all vectors (x_1, x_2, x_3, x_4) that are orthogonal (perpendicular) to both of these vectors:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

(3b) If u, v, w are three nonzero vectors in R^7, what are the possible dimensions of the subspace they span?

4. Suppose the 5 by 3 matrix U is in echelon form. Suppose it has $r = 3$ pivots.
(4a) What is the nullspace of U? (specific answer, not definition.)

(4b) What is the echelon form of the 10 by 3 matrix $B = \begin{bmatrix} U \\ 2U \end{bmatrix}$?

(4c) What is the echelon form of the matrix $C = \begin{bmatrix} U & U \\ U & 0 \end{bmatrix}$? Note: I got this wrong!

(4d) What is the rank of C?

(4e) What is the dimension of the nullspace of C^T?