# MULTIAGENT REINFORCEMENT LEARNING: ROLLOUT AND POLICY ITERATION

Dimitri P. Bertsekas
Arizona State University

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# Sources

## Based on material from my research monograph

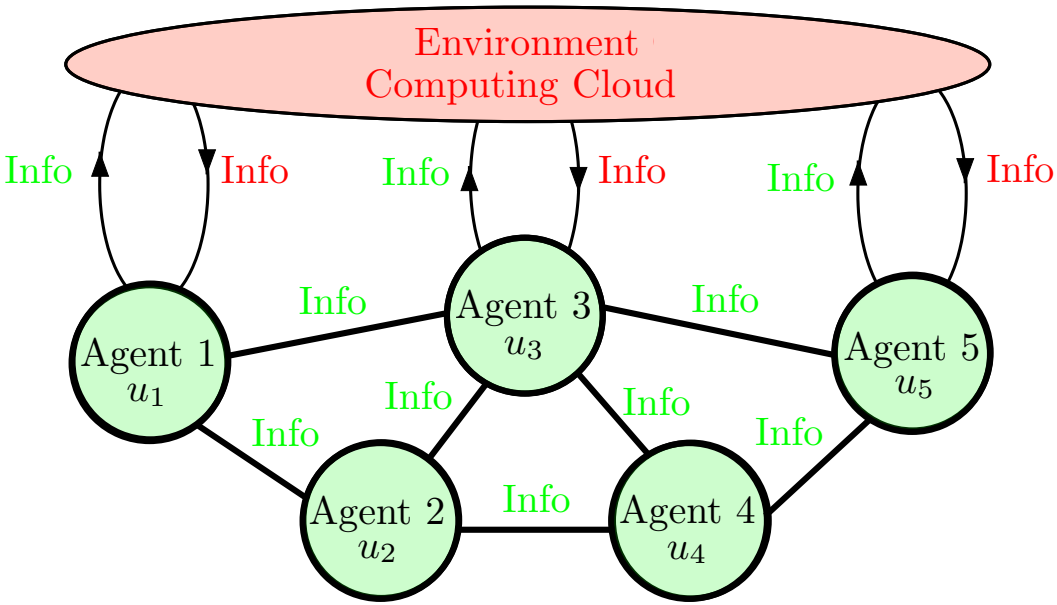
Rollout, Policy Iteration, and Distributed Reinforcement Learning, Athena Scientific, 2020

## Related research can be found at my website including:

- An overview paper to be published in IEEE/CAA J. of Automatica Sinica
- Several research papers and multiagent policy iteration, value iteration, and discrete deterministic optimization (2019-2020, ArXiv)
- A challenging multiagent POMDP repair problem paper, coauthored by S. Bhatacharya, S. Kailas, S. Badyal, and S. Gil (2020)

# Outline

# Multiagent Problems - A Very Old (1960s) and Well-Researched Field

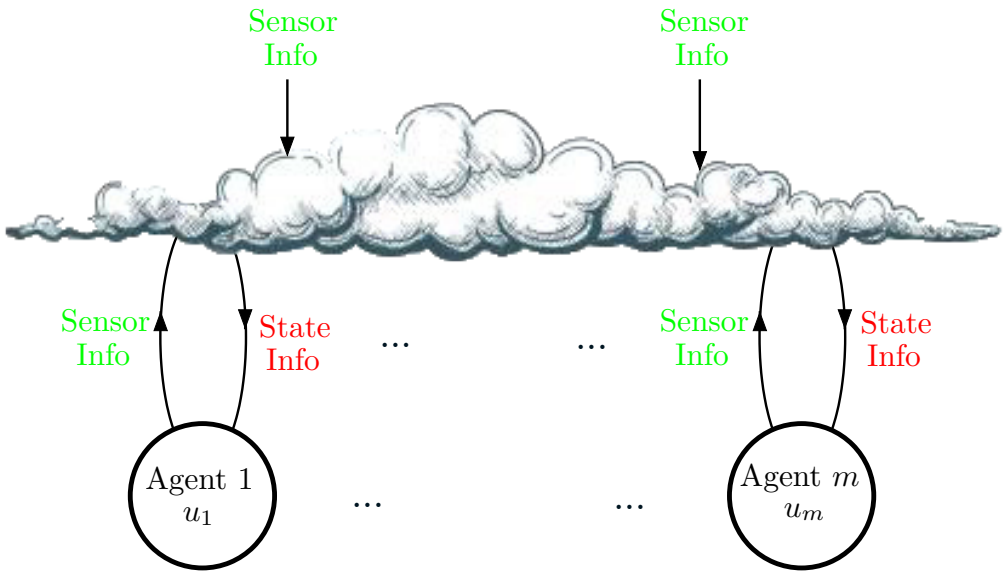



- Multiple agents collecting and sharing information selectively with each other and with an environment/computing cloud
- Agent $i$ applies decision $u_i$ sequentially in discrete time based on info received

## The major mathematical distinction between problem structures

- The classical information pattern: Agents are fully cooperative, fully sharing and never forgetting information. Can be treated by Dynamic Programming (DP)
- The nonclassical information pattern: Agents are partially sharing information, and may be antagonistic. HARD because it cannot be treated by DP

# Our Starting Point: A Classical Information Pattern ... but we will Generalize

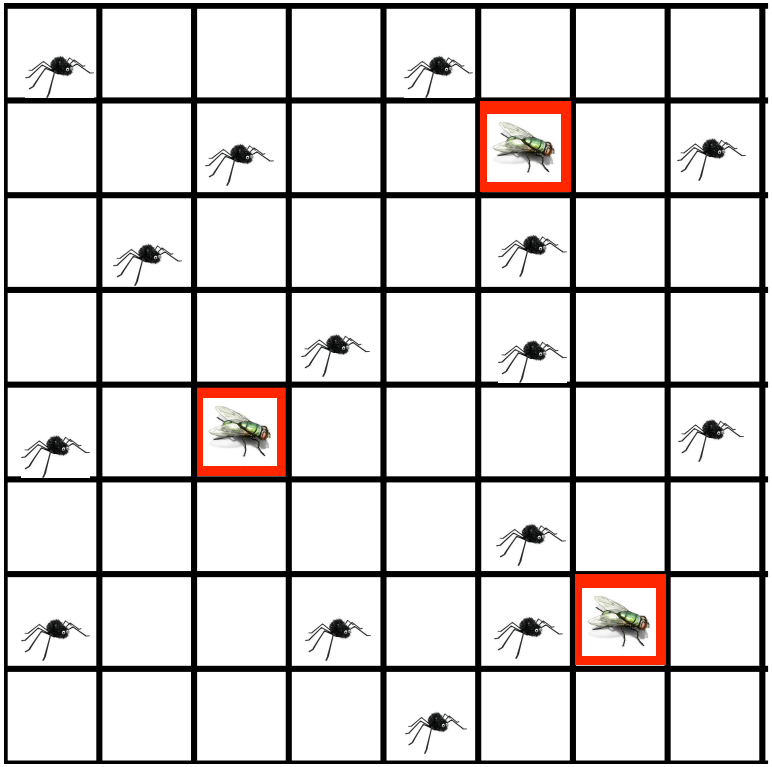



At each time, the agents have exact state info, and choose their controls as functions of the state

## Model: A discrete-time (possibly stochastic) system with state $x$ and control $u$

- Decision/control has $m$ components $u = (u_1, \ldots, u_m)$ corresponding to $m$ "agents"
- "Agents" is just a metaphor - the important math structure is $u = (u_1, \ldots, u_m)$
- The theoretical framework is DP. Our initial aim is faster computation
  - Deal with the exponential size of the search/control space
  - Be able to compute the agent controls in parallel (in the process we will deal in part with nonclassical info pattern issues)

# Spiders-and-Flies Example
# (e.g., Delivery, Maintenance, Search-and-Rescue, Firefighting)



15 spiders move in 4 directions with perfect vision

3 blind flies move randomly

- Objective is to catch the flies in minimum time
- At each time we must select one out of $\approx 5^{15}$ joint move choices
- We will reduce this to $5 \cdot 15 = 75$ choices (while maintaining good properties). Idea: Break down the control into a sequence of one-spider-at-a-time moves
- We will introduce “precomputed signaling/coordination" between the spiders, so the 15 spiders will choose moves in parallel (an extra speedup factor of 15)

# Nonclassical Information Pattern Approaches: A Summary

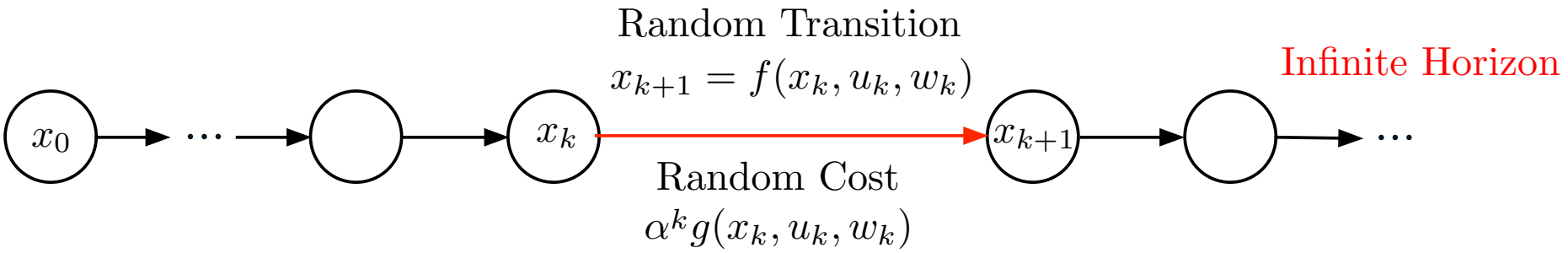
## Team theory/Decentralized control (also decentralized MDP and POMDP)

- Agents have common goals but do not fully share information; a nonclassical information pattern
- For example, some of the spiders can see some of the flies, but others cannot
- Notoriously difficult problems. Theory/algorithms here often try to exploit weak couplings between some of the agents

## A RL/policy gradient approach for nonclassical information patterns

- Forget about DP. Parametrize the agent policies in a way that is consistent with the information pattern
- Tune the parameters using neural networks and gradient descent (policy gradient methods)
- Advantage: Can deal with a nonclassical information pattern
- Drawback: Strictly off-line (and difficult) training (cannot adapt to on-line changes of problem data)
- No solid theory, lack of performance guarantees.

# For this Talk we Focus on Finite-State Infinite Horizon Problems




## Stationary system and cost accumulated over an infinite number of stages

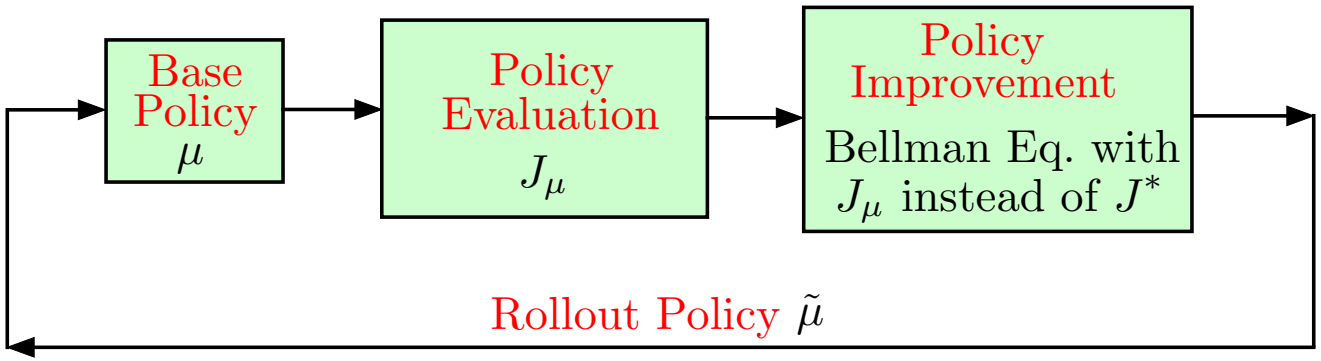
- System $x_{k+1} = f(x_k, u_k, w_k)$ with state $x_k$, $m$-component control $u_k$ ($w_k$: random)
- Policies $\mu = (\mu_1, \ldots, \mu_m)$ that map states $x$ to control components $\mu_i(x) \in U_i(x)$ for all $x$ and $i = 1, \ldots, m$
- Cost of stage $k$: $\alpha^k g\big(x_k, \mu_1(x_k), \ldots, \mu_m(x), w_k\big)$; $\alpha \in (0, 1]$ is the discount factor
- Cost of policy $\mu$

$$J_\mu(x_0) = \lim_{N\to\infty} E_{w_k}\left\{\sum_{k=0}^{N-1} \alpha^k g\big(x_k, \mu_1(x_k), \ldots, \mu_m(x), w_k\big)\right\}$$

- Optimal cost function $J^*(x_0) = \min_\mu J_\mu(x_0)$
- Optimality condition: Minimize the RHS of Bellman's equation

$$\mu^*(x) \in \arg\min_{(u_1,\ldots,u_m)} E_w\Big\{g(x, u_1, \ldots, u_m, w) + \alpha J^*\big(f(x, u_1, \ldots, u_m, w)\big)\Big\}$$

# Policy Iteration Algorithm




$$\tilde{\mu}(x) \in \arg \min_{(u_1,\ldots,u_m)} E_w\Big\{g(x,u_1,\ldots,u_m,w) + \alpha J_\mu\big(f(x,u_1,\ldots,u_m,w)\big)\Big\}$$

## Fundamental policy improvement property

$$J_{\tilde{\mu}}(x) \leq J_\mu(x), \quad \text{for all } x$$

## The rollout algorithm is a one-time policy iteration

It can be implemented on-line if values of $J_\mu$ are (approximately) available, through simulation (possibly in conjunction with a trained neural net)

## A second major advantage of the rollout algorithm: Robustness

If implemented on-line, it can adapt to variations of the problem data through on-line replanning

# Outline of Our Approach for Multiagent Problems

- We propose a policy iteration (PI) method with a new form of policy improvement, namely one-agent-at-a-time policy improvement
- Rollout is a single-iteration version of PI; but can be implemented on-line

## Extension to a nonclassical information pattern

- We use “guesses" to make up for missing information
- The “guesses" are precomputed, possibly through neural network training
- Subject of ongoing research

# A New Form of Policy Improvement Operation When $u = (u_1, \ldots, u_m)$

The standard rollout algorithm

$$\big(\tilde{\mu}_1(x), \ldots, \tilde{\mu}_m(x)\big) \in \arg \min_{(u_1, \ldots, u_m)} E_w \Big\{ g(x, u_1, \ldots, u_m, w) + \alpha J_\mu \big(f(x, u_1, \ldots, u_m, w)\big) \Big\}$$

has a search space with size that is exponential in $m$

## Proposed alternative: Multiagent rollout algorithm

Perform a sequence of $m$ successive minimizations, one-agent-at-a-time

$$\tilde{\mu}_1(x) \in \arg \min_{u_1} E_w \Big\{ g(x, u_1, \mu_2(x), \ldots, \mu_m(x), w) + \alpha J_\mu \big(f(x, u_1, \mu_2(x), \ldots, \mu_m(x), w)\big) \Big\}$$

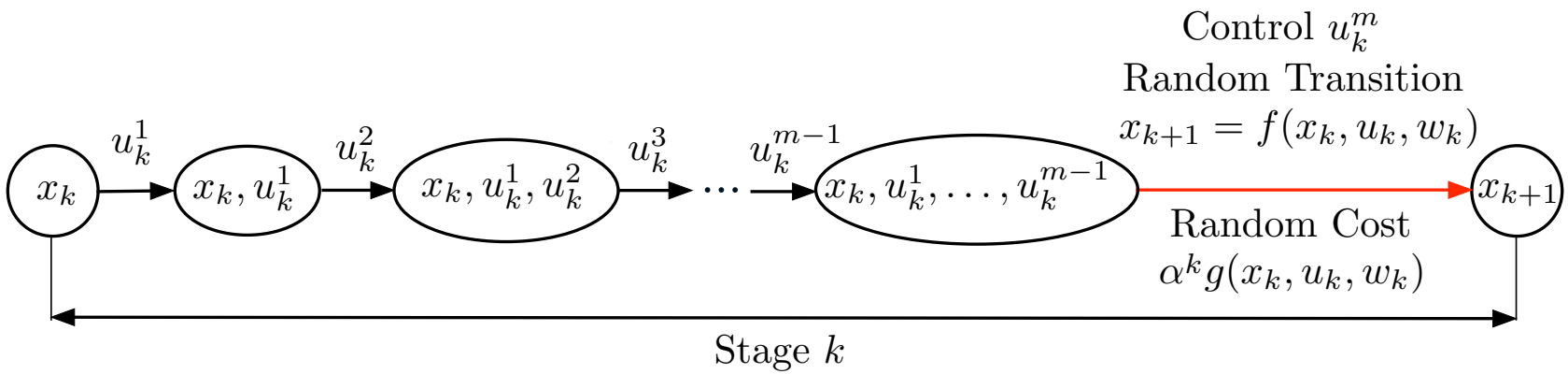
$$\tilde{\mu}_2(x) \in \arg \min_{u_2} E_w \Big\{ g(x, \tilde{\mu}_1(x), u_2, \mu_3(x) \ldots, \mu_m(x), w) + \alpha J_\mu \big(f(x, \tilde{\mu}_1(x), u_2, \mu_3(x), \ldots, \mu_m(x), w)\big)$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots$$

$$\tilde{\mu}_m(x) \in \arg \min_{u_m} E_w \Big\{ g(x, \tilde{\mu}_1(x), \tilde{\mu}_2(x), \ldots, \tilde{\mu}_{m-1}(x), u_m, w) + \alpha J_\mu \big(f(x, \tilde{\mu}_1(x), \tilde{\mu}_2(x), \ldots, \tilde{\mu}_{m-1}(x),$$

- Has a search space with size that is linear in $m$
- ENORMOUS SPEEDUP!

# Underlying Theory: Trading off Control and State Complexity (NDP book, 1996)




## An equivalent reformulation - "Unfolding" the control action

- The control space is simplified at the expense of $m-1$ additional layers of states, and corresponding $m-1$ cost functions

$$J^1(x_k, u_k^1), J^2(x_k, u_k^1, u_k^2), \ldots, J^{m-1}(x_k, u_k^1, \ldots, u_k^{m-1})$$

- Multiagent (one-agent-at-a-time) rollout is just standard rollout for the reformulated problem
- The increase in size of the state space does not adversely affect rollout
- Key theoretical fact: The cost improvement property is maintained
- Complexity reduction: The one-step lookahead branching factor is reduced from $n^m$ to $nm$, where $n$ is the number of possible choices for each component $u_k^i$

# Four Spiders and Two Flies: Illustration of Various Forms of Rollout

Base Policy - Greedy  Standard Rollout - All-at-once  Agent-by-Agent Rollout

Base policy: Move along the shortest path to the closest surviving fly (in the Manhattan distance metric)

## Time to catch the flies

- Base policy (each spider follows the shortest path): Capture time = 85
- Standard rollout (all spiders move at once, $4^5 = 625$ move choices): Capture time = 34
- Agent-by-agent rollout (spiders move one at a time, $4 \cdot 5 = 20$ move choices): Capture time = 34

# Comparing Standard with Multiagent Rollout/Policy Iteration

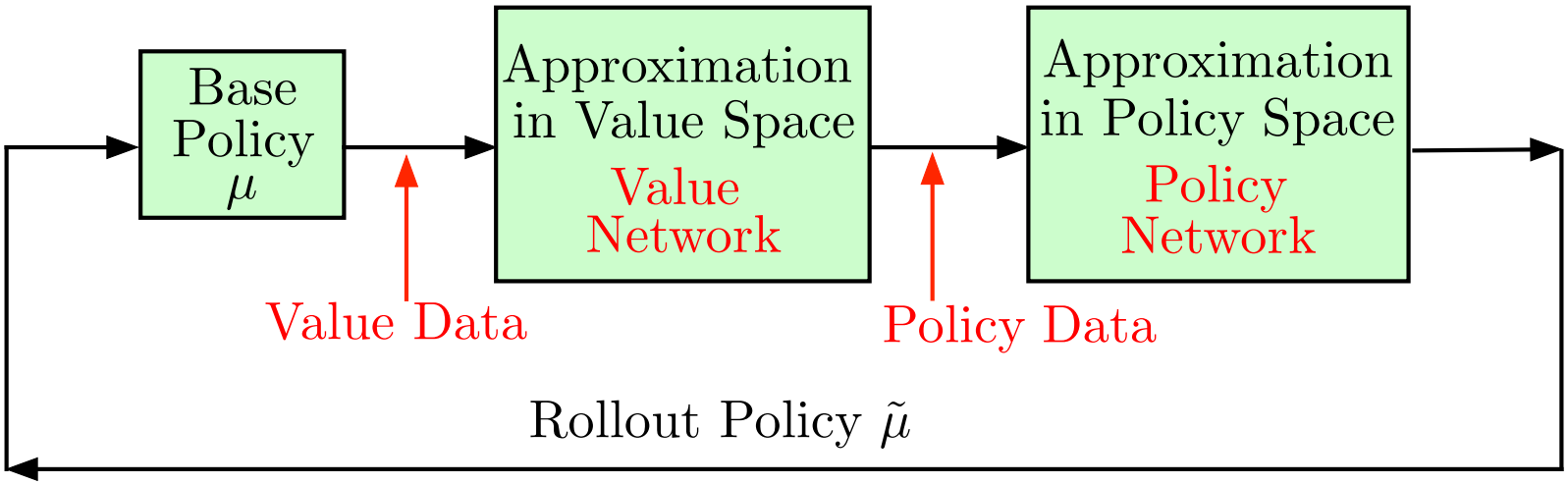
## Comparison of agent-by-agent and standard (all-agents-at-once) rollout

- They produce different rollout policies
- One may be better than the other
- ... BUT standard rollout requires intractable computation for even a modest number of agents
- We speculate that agent-by-agent rollout will often perform about as well as standard rollout

## Agent-by-agent PI: Uses agent-by-agent policy improvement

- Agent-by-agent PI stops at an agent-by-agent optimal policy ... which may not be optimal
- Convergence result: Agent-by-agent PI converges finitely to an agent-by-agent optimal policy
- Rate of convergence seems comparable to standard PI

# Approximate Policy Iteration with Agent-by-Agent Policy Improvement




- Approximate policy improvement property: With approximations, policy improvement holds approximately:

$$J_{\tilde{\mu}}(x) \leq J_{\mu}(x) + \epsilon, \qquad \text{for all } x$$

- If a single policy iteration is done (rollout), no need to train value and policy networks
- Multiple policy iterations can be done only with off-line training
- Many RL algorithms, including Alphazero, use schemes of this type (off-line PI plus on-line rollout)

# Parallelization of Agent-by-Agent Policy Improvement

One-agent-at-a-time policy improvement is an inherently serial computation. How can we parallelize it?

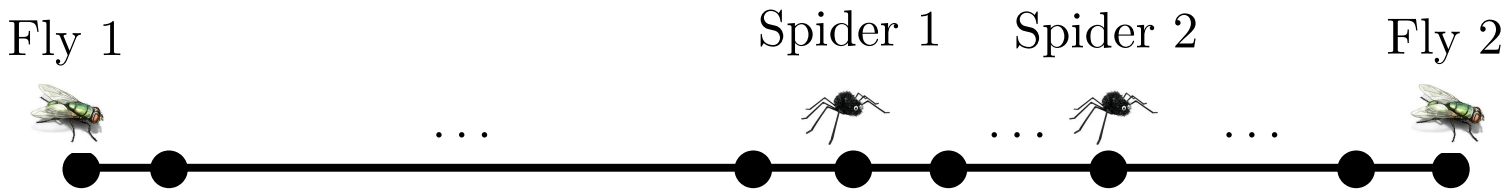
## Precomputed signaling

- Obstacle to parallelization: To compute the $k$th agent rollout control we need the rollout controls of the preceding agents $i < k$
- Signaling remedy: Use precomputed substitute "guesses" $\widehat{\mu}_i(x)$ in place of the preceding rollout controls $\tilde{\mu}_i(x)$

## Signaling possibilities

- Use the base policy controls for signaling $\widehat{\mu}_i(x) = \mu_i(x)$, $i = 1, \ldots, k-1$ (this may work poorly)
- Use a neural net representation of the rollout policy controls for signaling $\widehat{\mu}_i(x) \approx \tilde{\mu}_i(x)$, $i = 1, \ldots, k-1$ (this requires off-line computation)
- Other, problem-specific possibilities

# The Pitfall of Using the Base Policy for Signaling



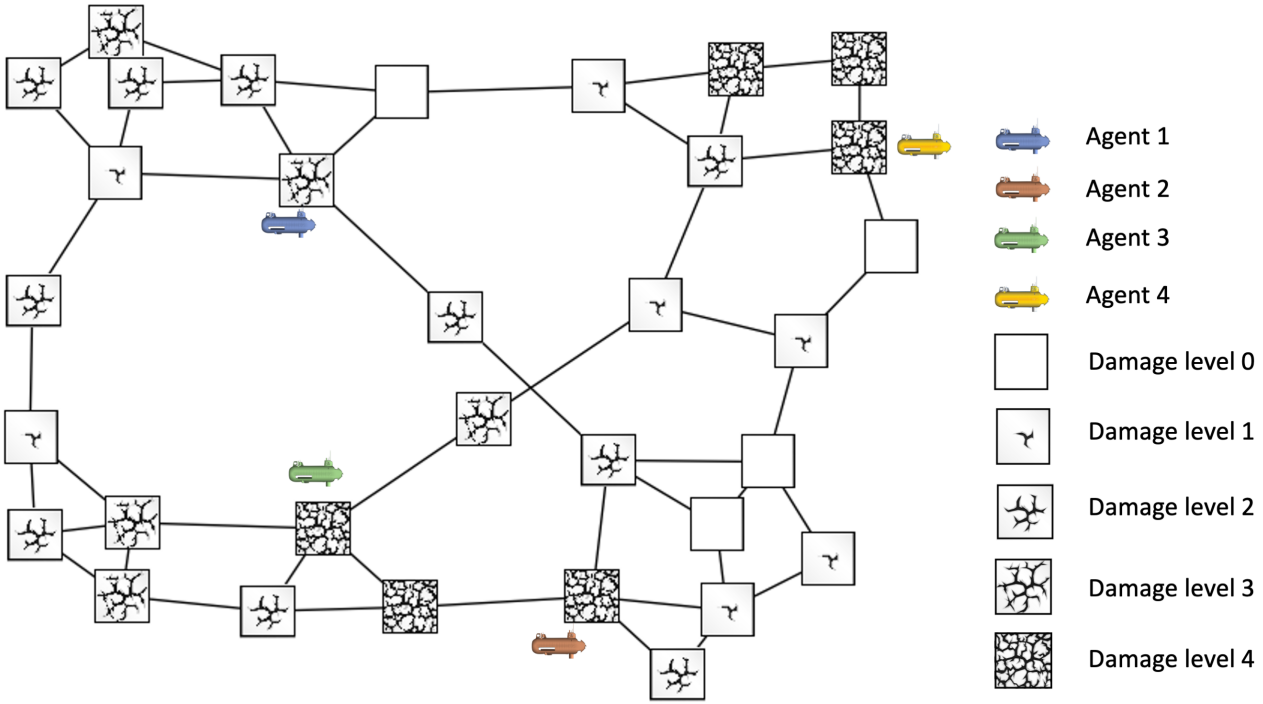

## Two spiders trying to catch two stationary flies in minimum time

- The spiders have perfect vision/perfect information
- Base policy for each fly: Move one step towards the closest surviving fly

## Performance of various algorithms

- Optimal policy: Splits the spiders towards their closest flies
- Standard rollout is optimal for all initial states
- Agent-by-agent rollout is also optimal for all initial states
- Agent-by-agent rollout with base policy signaling is optimal for most initial states, with A SIGNIFICANT EXCEPTION
- When the spiders start at the same location, the spiders oscillate and never catch the flies

# Multirobot Repair of a Network of Damaged Sites (BKBGB Paper)




- Damage level of each site is unknown, except when inspected. It deteriorates according to a known Markov chain unless the site is repaired
- Control choice of each robot: Inspect and repair (which takes one unit time), or inspect and move to a neighboring site
- State of the system: The set of robot locations, plus the belief state of the site damages (the joint probability distribution of the damage levels of the sites)
- Stage cost at each unrepaired site: Depends on the level of its damage

# Multirobot Repair in a Network of Damaged Sites Agents Start from the Same Location

Base Policy (Shortest Path)

Multiagent Rollout

Approx. Multiagent Rollout with Base Policy

Approx. Multiagent Rollout with Policy Net

## Cost comparisons

- Base policy cost: 5294 (30 steps)
- Multiagent rollout : 1124 (9 steps)
- Approx. Multiagent Rollout with base policy: 31109 (Never stops)
- Approx. Multiagent Rollout with neural network policy: 2763 (15 steps)

# Multirobot Repair in a Network of Damaged Sites Agents Start from Different Locations

Base Policy (Shortest Path)

Multiagent Rollout

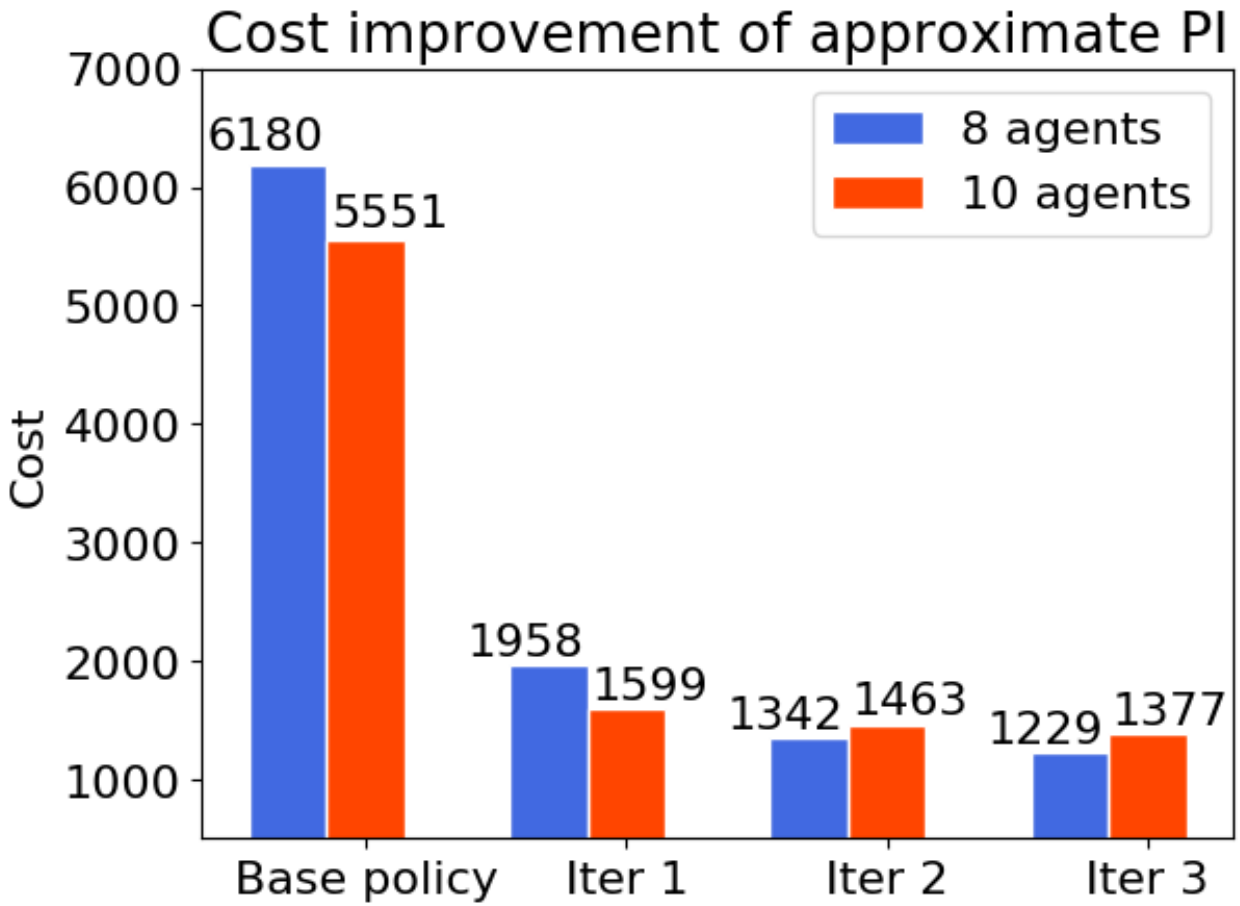
Approx. Multiagent Rollout with Base Policy

Approx. Multiagent Rollout with Policy Net

## Cost comparisons

- Base policy cost: 12945 (62 steps)
- Multiagent rollout : 5294 (19 steps)
- Approx. Multiagent Rollout with base policy: 6920 (25 steps)
- Approx. Multiagent Rollout with neural network policy: 7241 (21 steps)

# Approximate Policy Iteration with Policy Nets (BKBGB Paper)




- Recall that a policy network must be used to represent a policy generated by PI
- As a result the PI training must be done off-line
- Typical performance: Large cost improvement at first few iterations, which tails off and ends up in an oscillation as the number of generated policies increases

# Concluding Remarks

- We have focused on multiagent rollout and policy iteration
- These are approximation in value space methods that can be applied to very complex RL problems with multi-component controls
- We have introduced a simplified form of policy improvement for multiagent problems
- They offer a solid performance guarantee (performance improvement property)
- They have interesting theoretical properties (PI convergence to an agent-by-agent optimal policy)
- Our methods require a classical information pattern (key assumption is the sharing of perfect state info)
- They admit extensions to nonclassical information pattern problems through precomputed signaling policies
- Our methods compare very favorably on the multi-robot repair problem with existing methods (POMCP, DESPOT, MADDPG)
- Important research question: Can perfect state info be replaced by state estimates?
- A broad range of very challenging analytical and algorithmic questions lie ahead

Thank you!