Demonstration of the Casimir Force in the 0.6 to 6 μm Range

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The vacuum stress between closely spaced conducting surfaces, due to the modification of the zero-point fluctuations of the electromagnetic field, has been conclusively demonstrated. The measurement employed an electromechanical system based on a torsion pendulum. Agreement with theory at the level of 5% is obtained. [S0031-9007(96)02025-X]

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One of the most remarkable predictions of quantum electrodynamics (QED), obtained by Casimir in 1948, is that two parallel, closely spaced, conducting plates will be mutually attracted [1]. This attractive force is due to the exclusion of electromagnetic modes between the plates (as compared to free space) and has magnitude (per unit surface area A)

$$F(a)/A = \frac{\pi^2}{240}\frac{\hbar c}{a^4} = 0.016\,\frac{1}{a^4}\ \mathrm{dyn}\,(\mu m)^4/\mathrm{cm}^2, \quad (1)$$

where a is the plate separation; in principle, a QED effect can directly influence a macroscopic, classical, apparatus. In spite of the extensive theoretical attention this effect has received over the years (see [2,3] for recent reviews), there has been only one attempt at its measurement. This measurement, as reported by Sparnaay in 1958, showed an attractive force "not inconsistent with" the prediction given by Eq. (1), but with effectively 100% uncertainty [4]. A closely related effect, the attraction of a neutral atom to a conducting plate, has been recently measured [5]; good agreement with theory was found.

The Casimir force is closely related to the van der Waals attraction between dielectric bodies. Formally, Eq. (1) is obtained by letting the dielectric constant ϵ in the Lifshitz theory [6] approach infinity, which is an appropriate description for a conducting material. However, in practical terms, the Casimir and van der Waals forces are quite different; the van der Waals force is always attractive, whereas the sign of the Casimir force is geometry dependent. For example, if a thin spherical conducting shell is cut in half, the two hemispheres will experience a mutual *repulsive* force [7]. These points are discussed in Refs. [2,3]. A number of experimental measurements of short-range forces between dielectric bodies of various forms have been performed; see Ref. [2] for a review.

For our measurement of the Casimir force, the conductors were in the form of a flat plate and a sphere. Our first attempts at measurements using parallel plates were unsuccessful; this is because it is very difficult to maintain parallelism at the requisite accuracy (10^{-5} rad for 1 cm diameter plates). There is no issue of parallelism when one plate has a spherical surface; geometrically, the system is described by the separation at the point of closest approach. However, when one plate is spherical, Eq. (1) must be modified; the force for this geometry is simply obtained by the use of the so-called proximity force theorem (PFT) [8], which in the present case reduces to $F = 2\pi RE$ where R is the radius of curvature of the spherical surface, and E is the potential energy per unit surface area which gives rise to the force of attraction between flat plates. Thus, the magnitude of the Casimir force between a sphere and a flat surface is given by

$$F_c(a) = 2\pi R\left(\frac{1}{3}\frac{\pi^2}{240}\frac{\hbar c}{a^3}\right). \quad (2)$$

and the result is independent of the plate area.

There are at least two corrections to the Casimir force. The first is the effect due to the finite temperature $T \approx 300$ K; this correction has an illustrious history as discussed by Schwinger *et al.* [9]; the thermal corrections for the Casimir force and van der Waals force are different, and are properly derived for the case of conducting plates in Refs. [9–11]. Taking the results of Brown and Maclay [11], the surface energy is given by $E = aT^{00}$, where T^{00} is the (volume) energy density. Using the PFT and Eq. (20a) of [11], the total magnitude of the Casimir force is

$$F_c^T(a) = F_c(a)\left(1 + \frac{720}{\pi^2} f(\xi)\right), \quad (3)$$

where $\xi = kTa/\hbar c = 0.126a\ \mu\mathrm{m}^{-1}$ at $T = 300$ K (k is Boltzmann's constant) and

$$f(\xi) \approx \begin{cases} (\xi^3/2\pi)\zeta(3) - (\xi^4\pi^2/45), & \text{for } \xi \le 1/2\,, \\ (\xi/8\pi)\zeta(3) - (\pi^2/720), & \text{for } \xi > 1/2\,, \end{cases} \quad (4)$$

where $\zeta(3) = 1.202\ldots$. It is interesting to note that in the large a limit, the correction is independent of $\hbar c$ and has the appearance of a classical effect; this is analogous to the Rayleigh-Jeans limit of the black body spectrum.

The second correction, obtained by Schwinger *et al.* [9], is due to the finite conductivity of the plates (modified by the use of the PFT to the case where one plate is spherical);

$$F_c'(a) = F_c(a)\left(1 + \frac{4c}{a\omega_p}\right), \quad (5)$$

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where ω_p is the plasma frequency for the conductor, and it was assumed that the effective electric susceptibility is $\epsilon(\omega) = 1 - (\omega_p/\omega)^2$.

A schematic of the apparatus used in our measurement is shown in Fig. 1, and details of the torsion pendulum are shown in Fig. 2. The Casimir force plates comprise a 2.54 cm diam, 0.5 cm thick quartz optical flat, and a spherical lens with radius of curvature 11.3 ± 0.1 cm and diameter 4 cm; each was coated (by evaporation) with a continuous layer of Cu of thickness 0.5 μm, on all surfaces. A layer of Au was then evaporated (0.5 μm thick) onto the faces which were subsequently brought together. As shown in Fig. 1, the flat electrode was mounted on one arm of the torsion pendulum, while the spherical electrode was placed on a micropositioning assembly. The adjustment screws and piezoelectric stack translators (PZTs) form a tripod. A vacuum of order 10^{-4} torr was maintained (to eliminate viscous effects) by a small oil (Fomblin) diffusion pump; adjustment rods passed through simple vacuum rotary feedthroughs and allowed coarse adjustment of the plate position, with about 0.5 μm accuracy.

A feedback system was used to keep the torsion pendulum angle fixed; as shown in Fig. 1, two "compensator plates" form a capacitor with respect to the pendulum body. An ac bridge circuit was used to determine whether the two capacitances are equal; any unbalance was detected by a phase-sensitive detector which provides an error signal to an integral-plus-proportional feedback circuit. A dc correction voltage was applied to the compensators, as required to keep the torsion pendulum angle fixed. In addition, a constant dc voltage of $V_0 = 7.5$ V was applied to the compensators in order to linearize the effect of the small correction voltage δV [the force on the pendulum due to one compensator is $F = (V^2/2)dC/dx \approx (V_0^2/2 \pm V_0\delta V)dC/dx$, and the net force from both is $F = 2V_0\delta V dC/dx$, where dC/dx is the magnitude of the change in compensator-pendulum capacitance as the gap size x is varied]. Since the feedback only affects the torsional mode, a strong magnet (2.5 kG surface field) was used to overdamp all vibrational modes of the pendulum system. The angle voltage signal from the PSD had sensitivity of 48 mV/μm displacement of the Casimir electrode (1 μm displacement corresponds to an angle of 2×10^{-5} rad). The angular fluctuations were consistent with the expected thermal noise [12]; $\delta\theta_{\rm rms} = \sqrt{kT/\alpha} \approx 1$ μrad, where $\alpha = 4.8$ dyn/rad is the torsion constant for the tungsten fiber. A microrotation stage allowed turning the fiber to set δV $= 0$. Before applying the vacuum, the fiber was annealed, with the pendulum hanging, by passing about 500 mA through it; while the current was flowing, the fiber length increased by about 1%. The drift in apparent force, over one month, was less than 2×10^{-3} dyn.

The Casimir force was measured by simply stepping the voltage applied to the PZTs up and down through 16 discrete and constant steps, and at each step, measuring the restoring force, implied by a change in δV, required to keep the pendulum angle fixed. The maximum displacement at 92 V was 12.3 μm; the relative displacement

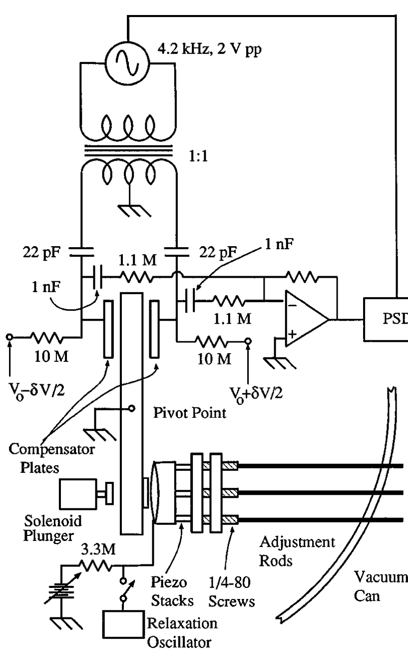

FIG. 1. Schematic of the apparatus. The vacuum vessel dimensions are 55 cm diam by 110 cm tall. The solenoid activated plunger was used to press the plates gently together (during alignment); after such pressing, the plates could be brought much closer.

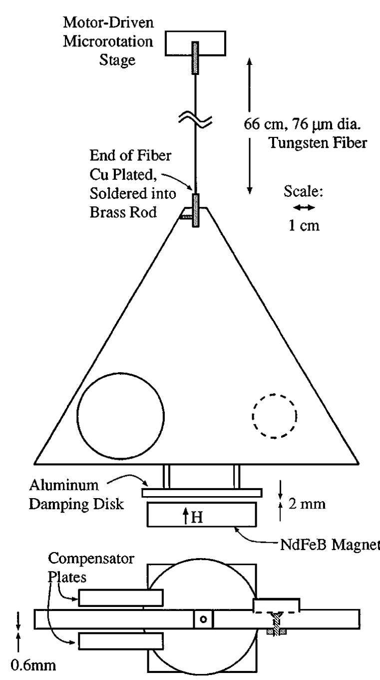

FIG. 2. Details of the pendulum. The body has total mass 397 g. The ends of the W fiber were plated with a Cu cyanide solution; the fiber ends were bent into hairpins of 1 cm length and then soldered into a 0.5 mm diam, 7 mm deep holes in the brass rods. Flat-head screws were glued to the backs of the plates; a spring and nut held the plates firmly against their supports, and ensured good electrical contact.

as a function of the sixteen steps (up and down separately because of considerable hysteresis) was measured to 0.01 μm accuracy by use of a laser interferometer, giving 32 values a_i for the relative plate position. The average displacement per 5.75 volt step was about 0.75 μm. An up/down sweep sequence typically took 25 min; after each step, no data were taken for 40 s (about three system response time constants) to ensure that the feedback system was in equilibrium, after which δV was averaged for 7 s.

The system calibration was obtained through electrical measurements based on the variation of the capacitance between the Casimir plates as a function of separation. A simple relaxation oscillator that only required 30 mV rms to be applied between the plates was used to determine the capacitance variation; the oscillator frequency was carefully calibrated by a precision variable capacitor in parallel with the plates, and was typically 11.24 Hz/pF. Using the PZTs to change the plate separation a, $dC/da \approx 0.73$ pF/μm, with 1% accuracy, at the plate separation where the calibration was performed (about 10 μm). Next, a potential difference $V = 300$ mV was applied between the plates; this generates a force $F = (V^2/2)dC/da$; the calibration was determined by the change in δV from the feedback circuit, and is 0.0420 dyne/volt, with 1% accuracy. This calibration is in good agreement with (less accurate) direct measurement of the physical dimensions of the compensator plates and their spacing.

With the Casimir plates separated but externally shorted together, there was an apparent shockingly large potential of 430 mV; there are roughly 40 separate electrical connections in this loop and a potential this large is consistent with what is expected for the various metallic contacts. This potential was easily canceled by setting an applied voltage between the plates to give a minimum δV; this applied voltage was taken as "zero" in regard to the calibration. The contact potential drifted by less than 10 mV over the one month of operation of the experiment, and was independent of the region of the plates where the measurement was performed (this was varied by tilting the spherical plate with the adjustment screws).

The PZTs give very accurate and reproducible *relative* changes in the plate separation; the absolute separation was determined by measuring the residual electrical attraction between the plates as a function of separation (the contact potential was intentionally not perfectly canceled). The electric force as a function of separation can be obtained by use of the PFT. For two flat parallel plates separated by a distance a with a potential V between them, the potential energy per unit area is $E = \frac{1}{2}(C/A)V^2 = \frac{1}{2}V^2\epsilon_0/a$. Applying the PFT theorem gives the electric force as

$$F_e(a) = \pi\epsilon_0 RV^2/a \rightarrow C(a) = 2\pi\epsilon_0 R \ln a + C_0\,, \quad (6)$$

where $C(a)$ is the capacitance between the plates. This result agrees with the exact solutions for the force and the capacitance between a sphere and a plane, in the limit $a/R \rightarrow 0$ [13]. Thus, the electric force between the plates varies as $1/a$.

Each up/down sweep cycle was fitted to a function of form

$$F^m(i) = F_c^T(a_i + a_0) + \frac{\beta}{a_i + a_0} + b\,, \quad (7)$$

where $F^m(i)$ is the measured force at the ith step, a_0 is a fit parameter which gives the absolute plate separation, and b is a constant; the points of closest approach (closer than 2 μm) are not used in this fit; this allows the true Casimir force, which is insignificant at large distances, to be discriminated from the electrical force, as shown in Fig. 3. Next, the absolute position is assigned to $F^m(i)$, and a_i is set to $a_i + a_0$. Finally, the electrical force is subtracted, giving

$$F_c^m(a_i) = F(a_i) - \frac{\beta}{a_i} - b\,, \quad (8)$$

where $F_c^m(a_i)$ is the measured residual force (hopefully the Casimir force). The results of the process, and a direct comparison with the expected Casimir force with no adjustable parameters, is shown in Fig. 3. Typically, the Casimir force had magnitude of at least 20% of the electrical force at the point of closest approach. There was no evidence for a $1/a^2$ force in any of the data; a force of this form could result from patch effects on the surfaces.

The uncertainty in a_0 was normally less than 0.1 μm. Typically, a_0 drifted by about 0.1 μm per up/down sweep; this allowed a wide variety of closest approaches to be investigated, even though the discrete step size was rather

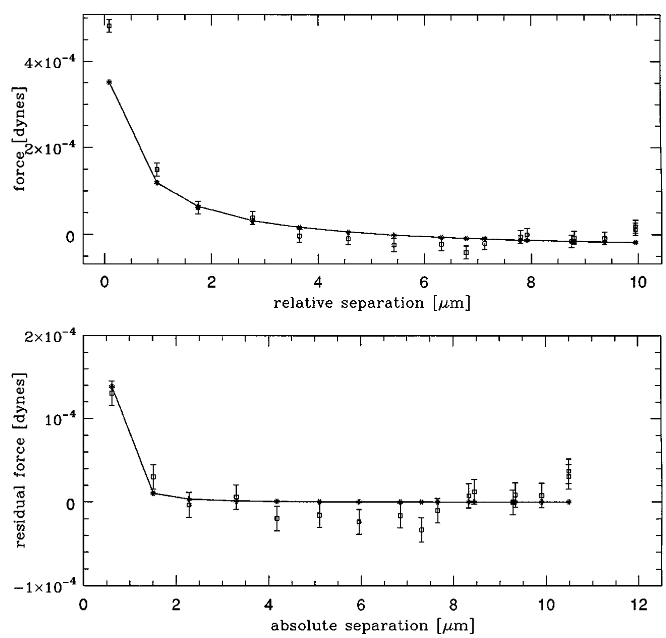

FIG. 3. Top: Measured force as a function of relative position and least-squares fit to $F_c^T(a_i + a_0) + \beta/(a_i + a_0) + b$; the two points at closest approach were not used in the fit. Bottom: Measured force with electric contribution subtracted; the points connected by lines are as expected from the Casimir force.

large. This drift was the result of a variety of environmental factors, most notably temperature variations. Roughly 10% of the up/down sweeps were rejected because anomalously large drift resulted in an unsatisfactory convergence for the fit, as evidenced by an anomalously large χ^2, a nonphysical value for a_0, and an inconsistent result for β which was quite constant. Also, those sweeps where the net change in δV between start and finish corresponded to a force greater than 4×10^{-5} dyn were rejected; in all, the final data set comprises 216 up/down sweeps. Quite often, as the absolute separation drifted, the plates would contact before the end of a complete up sweep. The step at which this occurred could be unambiguously determined by a sudden jump in the feedback signal. Roughly eight steps on the down sweep had to be rejected because after such a large perturbation, the feedback system required several minutes to reestablish equilibrium.

Assuming that the functional form for the Casimir force is correct, its magnitude was determined by using linear least squares to determine a parameter δ for each sweep such that

$$F_c^m(a_i) = (1 + \delta)F_c^T(a_i) + b'. \quad (9)$$

In this context, b' should be zero, and for the complete data set, $b' < 5 \times 10^{-7}$ dyn (95% confidence level). The average over the 216 sweeps gives $\delta = 0.01 \pm 0.05$, and this is taken as the degree of precision of the measurement. There was no evidence for any variation of δ depending on the region of the plates used for the measurement.

The most striking demonstration of the Casimir force is given in Fig. 4. The agreement with theory, with no adjustable parameters, is excellent. It should be noted that the closest approach is about 0.6 μm; this limit could be caused by either dirt on the surfaces, or by an instability of the feedback system. The Casimir force is nonlinear and increases rapidly at distances less than 0.5 μm. With the plates separated by 10 μm, the feedback loop became unstable when a 700 mV potential difference between the plates was applied; the change in force with distance (the effective spring constant) in this case is $dF_e/da = 1.5 \times 10^{-3}$ dyn/μm, which is equal to dF_c/da at $a = 0.5$ μm.

In conclusion, we have given an unambiguous demonstration of the Casimir force with accuracy of order 5%. Our data is not of sufficient accuracy to demonstrate the finite temperature correction, as shown in Fig. 4(b). Also, given a plasma frequency for Au of order $\omega_p/2\pi \approx 6 \times 10^{14}$ Hz, Eq. (5) gives a correction of order 20% at the closest spacings; our data does not support such a deviation. However, the simple frequency dependence of the electrical susceptibility used in the derivation of Eq. (5) is not correct for Au, the index of refraction of which has a large imaginary component above the plasma frequency; a rough estimate using the tabulated complex index [14] limits the conductivity correction as no larger than 3%, which is consistent with our results [15].

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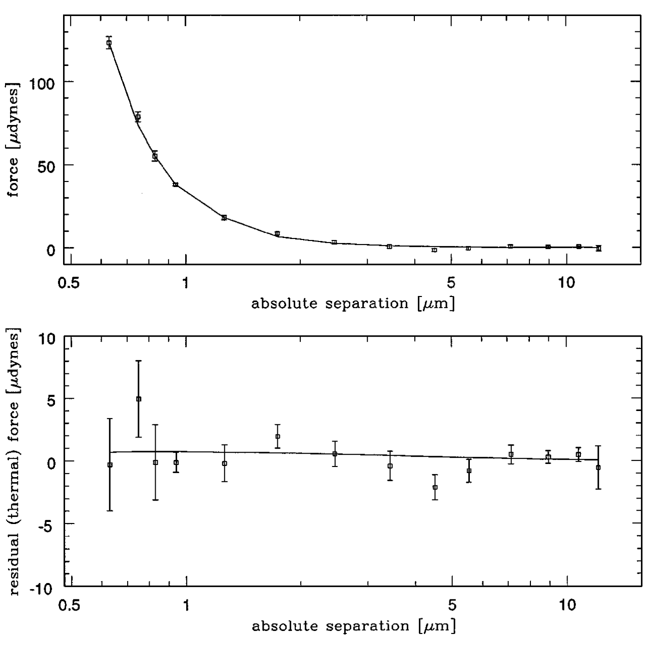

FIG. 4. Top: All data with electric force subtracted, averaged into bins (of varying width), compared to the expected Casimir force for a 11.3 cm spherical plate. Bottom: Theoretical Casimir force, without the thermal correction, subtracted from top plot; the solid line shows the expected residuals.

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[1] H. B. G. Casimir, Koninkl. Ned. Adak. Wetenschap. Proc. **51**, 793 (1948).
[2] E. Elizalde and A. Romeo, Am. J. Phys. **59**, 711 (1991).
[3] V. M. Mostepanenko and N. N. Trunov, Sov. Phys. Usp. **31**, 965 (1988).
[4] M. J. Sparnaay, Physica (Utrecht) **24**, 751 (1958).
[5] C. I. Sukenik, M. G. Boshier, D. Cho, V. Sangdohar, and E. A. Hinds, Phys. Rev. Lett. **70**, 560 (1993).
[6] E. M. Lifshitz, Sov. Phys. JETP **2**, 73 (1956).
[7] T. H. Boyer, Phys. Rev. **174**, 1764 (1968).
[8] J. Blocki, J. Randrup, W. J. Swiatecki, and C. F. Tsang, Ann. Phys. (N.Y.) **105**, 427 (1977).
[9] J. Schwinger, L. L. DeRaad, Jr., and K. A. Milton, Ann. Phys. (N.Y.) **115**, 1 (1978).
[10] J. Mehra, Physica (Utrecht) **37**, 145 (1967).
[11] L. S. Brown and G. J. Maclay, Phys. Rev. **184**, 1272 (1969).
[12] G. Ising, Philos. Mag. **1**, 827 (1926).
[13] W. R. Smythe, *Static and Dynamic Electricity* (McGraw-Hill, New York, 1950), pp. 121–122.
[14] *CRC Handbook of Chemistry and Physics, 76th Ed.* (CRC Press, Boca Raton, 1995), pp. 12--130.
[15] S. Hacyan, R. Jauregui, F. Soto, and C. Villarreal, J. Phys. A **23**, 2401 (1990).