Efficiency Loss in Cournot Oligopolies

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4. **Efficiency Loss with Taxation**
Consider a market for a single homogeneous good with \( N \) suppliers. Supplier \( n \in \{1, 2, \ldots, N\} \) has a cost function \( C_n \).

The inverse demand function \( p : [0, \infty) \to [0, \infty) \), which maps the total supply into price.

Suppose that supplier \( n \) produces \( x_n \) amount of good, and the price will be \( p(X) \), where \( X = \sum_{n=1}^{N} x_n \).

The payoff (profit) of supplier \( n \) is

\[
\pi_n(x_n, x_{-n}) = x_n p(X) - C_n(x_n).
\]
Competitive equilibrium

A strategy profile \( \{x_1, x_2, \ldots, x_N\} \) is a **competitive equilibrium**, if

\[
x_n\mu - C_n(x_n) \geq x\mu - C_n(x), \quad \forall x \geq 0, \quad \forall n \in \{1, 2, \ldots, N\},
\]

where \( \mu = p\left(\sum_{n=1}^{N} x_n\right) \).

**Oligopoly Theory** (Chapter 2.4), by Friedman (1983)

Is the Cournot equilibrium close, in some reasonable sense, to the competitive equilibrium?
Efficiency Loss


Social Welfare in a Cournot oligopoly

One price-taking (representative) consumer maximizes her own payoff $U(X) - p(X)X$.

We have $U'(X) = p(X)$, and therefore $U(X) = \int_0^X p(q) dq$. 
Aggregate consumer utility: \( U(X) = \int_0^X p(q) dq. \)

Consumer surplus: \( \int_0^X p(q) dq - p(X)X. \)
Supplier profit: \( p(X)X - \sum_{n=1}^{N} C_n(x_n) \).

Social welfare: \( \int_0^X p(q) dq - \sum_{n=1}^{N} C_n(x_n) \).
Definition

The **efficiency** of a nonnegative vector $\mathbf{x} = (x_1, \ldots, x_N)$ is defined as

$$
\gamma(\mathbf{x}) = \frac{\int_0^{\sum_{n=1}^N x_n} p(q) dq - \sum_{n=1}^N C(x_n)}{\int_0^{\sum_{n=1}^N x_n^S} p(q) dq - \sum_{n=1}^N C(x_n^S)},
$$

where $(x_1^S, \ldots, x_N^S)$ is an optimal solution to the social planner’s problem:

maximize $\int_0^{\sum_{n=1}^N x_n} p(q) dq - \sum_{n=1}^N C_n(x_n)$

subject to $x_n \geq 0$, $n = 1, 2, \ldots, N$. 

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Efficiency Loss with Taxation
Anderson and Renault (2003) study the relation between consumer surplus, producer surplus, and the aggregate social welfare achieved at a Cournot equilibrium.

Kluberg and Perakis (2008) compare the social welfare under Cournot competition to the maximum possible, when the price demand relationship is linear.

Johari and Tsitsiklis (2005) establish a $2/3$ efficiency lower bound, when the inverse demand function is affine.
Assumptions

On Cost Functions
For any \( n \), the cost function \( C_n(\cdot) \) is convex and nondecreasing on \([0, \infty)\), and continuously differentiable on \((0, \infty)\), with \( C_n(0) = 0 \).

On the Inverse Demand Function
The inverse demand function \( p : [0, \infty) \rightarrow [0, \infty) \) is continuous, nonnegative, nonincreasing and convex with \( p(0) > 0 \).

We also assume that
- There exists some \( R > 0 \) such that \( p(R) \leq \min_n \{ C'_n(0) \} \).
- The price at zero supply is larger than the minimum marginal cost, i.e., \( p(0) > \min_n \{ C'_n(0) \} \).
1. The output (aggregate supply) at a Cournot equilibrium is positive.

2. The social welfare achieved at a Cournot equilibrium is positive.

3. All social optima yield the same price.

4. Let $x$ and $x^S$ be a Cournot equilibrium and a socially optimum, respectively. If $p(X) \neq p(X^S)$, then the output at a Cournot equilibrium is less than at a social optimum, $X < X^S$.

5. If $p(X) = p(X^S)$, the Cournot equilibrium is efficient, i.e., $\gamma(x) = 1$. 
The worst cost functions are linear

Given a Cournot equilibrium \( \mathbf{x} = (x_1, \ldots, x_N) \), its efficiency cannot increase if we replace every supplier \( n \)'s cost function by \( \overline{C}_n(x) = \alpha_n x \), where \( \alpha_n = C'_n(x_n) \).
Piece-wise Linear Inverse Demand Functions
The efficiency of $x$ in the original model, with the inverse demand function $p(\cdot)$, is

$$
\gamma(x) = \frac{A + D - \sum_{n=1}^{N} C_n(x_n)}{A + B + D - \sum_{n=1}^{N} C_n(x_n^S)} \leq 1.
$$

$x^S$ remains socially optimal in the modified model with the piece-wise linear inverse demand curve $p^0$.

In the modified model with $p^0(\cdot)$,

$$
\gamma^0(x) = \frac{A - \sum_{n=1}^{N} C_n(x_n)}{A + B + C - \sum_{n=1}^{N} C_n(x_n^S)} \leq \gamma(x)
$$
In the modified model with \( p^0(\cdot) \), an efficiency lower bound can be obtained by solving the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \int_0^X p^0(q) dq - \sum_{n=1}^N \alpha_n x_n \\
& \quad \frac{\int_0^{X^S} p^0(q) dq - \sum_{n=1}^N \min_n \{\alpha_n\} X^S}{\int_0^X p^0(q) dq - \sum_{n=1}^N \alpha_n x_n} \\
\text{subject to} & \quad \alpha_n = p^0(X) - x_n c, \quad \text{if } x_n > 0 \\
& \quad \alpha_n \geq p^0(X), \quad \text{if } x_n = 0 \\
& \quad x_n \geq 0, \quad n = 1, 2, \ldots, N, \\
& \quad \sum_{n=1}^N x_n = X > 0,
\end{align*}
\]

where \( \alpha_n \) is the slope of supplier \( n \)'s cost function, and \( x^S \) is an optimal solution to the social planner’s problem.
Theorem

Let \( x \) and \( x^S \) be a Cournot equilibrium and a social optimum, respectively. We have

1. If \( p(X) = p(X^S) \), then \( \gamma(x) = 1 \).

2. If \( p(X) \neq p(X^S) \), let \( c = \left| p'(X) \right| \), \( d = \left| (p(X^S) - p(X))/(X^S - X) \right| \) and \( \bar{c} = c/d \). We have

\[
\gamma(x) \geq f(\bar{c}) = \frac{1/2 + \phi^2}{\bar{c}\phi^2/2 + \phi + 1/2}, \quad \bar{c} \geq 1,
\]

where

\[
\phi = \min \left\{ \frac{\bar{c} - 2 + \sqrt{\bar{c}^2 - 4\bar{c} + 12}}{4}, 1 \right\}.
\]

3. The bound is tight at \( \bar{c} = 1 \), e.g., when the inverse demand function is affine.
Illustration of the efficiency lower bound

\[
\frac{c}{d} \quad f\left(\frac{c}{d}\right)
\]

(1, 2/3)
Corollary

Let

\[ s = \inf \{ q : p(q) = \min_n C'_n(0) \}, \quad t = \inf \left\{ q : \min_n C'_n(q) \geq p(q) + q\partial_+ p(q) \right\} \]

If \( \partial_- p(s) < 0 \), then the efficiency of a Cournot equilibrium is at least \( f(\partial_+ p(t)/\partial_- p(s)) \).

Proof Sketch

- The output at any Cournot equilibrium is at least \( t : c \leq \partial_+ p(t) \).
- There exists a social optimum with an output no more than \( s : d \geq \partial_- p(s) \).
- \( c/d \leq \partial_+ p(t)/\partial_- p(s) \), and \( f(\cdot) \) is decreasing.
For the following group of inverse demand functions (eq.6, Bulow and Peiderer (1983))

\[ p(q) = \alpha - \beta \log q, \quad \alpha, \beta > 0, \quad 0 < q < \exp(\alpha/\beta). \]

We have

\[ s = \exp \left( \frac{\alpha - \chi}{\beta} \right), \quad t = \exp \left( \frac{(\alpha - \beta - \chi)/\beta}{\beta} \right), \]

and then,

\[ \gamma(x) \geq f \left( \frac{\exp ((\alpha - \chi)/\beta)}{\exp ((\alpha - \beta - \chi)/\beta)} \right) = f (\exp (1)) \geq 0.5237. \]
Consider a group of constant elasticity demand curves (eq.4, Bulow and Peiderer (1983))

\[ p(q) = \alpha q^{-\beta}, \quad 0 < \alpha, \quad 0 < \beta < 1, \quad 0 \leq q. \]

We have

\[ s = \left( \frac{\chi}{\alpha} \right)^{-1/\beta}, \quad t = \left( \frac{\chi}{\alpha(1-\beta)} \right)^{-1/\beta}, \quad \text{and then} \]

\[ \gamma(x) \geq f \left( -\frac{\alpha \beta t^{-\beta-1}}{-\alpha \beta s^{-\beta-1}} \right) = f \left( (1 - \beta)^{\frac{-\beta-1}{\beta}} \right). \]
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4 Efficiency Loss with Taxation
Unit Tax vs Ad Valorem Tax

**Unit Tax**

Under a unit (specific) tax rate $t$, supplier $n$'s profit is

$$p(X)x_n - C_n(x_n) - tx_n.$$ 

Tax revenue: $tX$.

**Ad Valorem Tax**

Under an ad valorem tax rate $\tau$, supplier $n$'s profit is

$$(1 - \tau)p(X)x_n - C_n(x_n).$$

Tax revenue: $\tau p(X)X$. 
Social welfare = Consumer surplus + Supplier profit + Tax Revenue.
\[ \text{Social welfare} = \int_0^X p(q) dq - \sum_{n=1}^N C_n(x_n). \]

Related Work

1. In a monopoly market, Ad Valorem tax Pareto dominates unit tax (Wicksell 1896, Suits and Musgrave 1953, Skeath and Trandel 1994).

2. In a Cournot oligopoly, ad valorem tax does not always Pareto dominate unit tax (Skeath and Trandel 1994).

3. Ad Valorem tax welfare dominates unit tax in a Cournot oligopoly, when all suppliers are symmetric (Delipalla and Keen 1992), or when supplier cost functions are linear (Anderson et. al. 2000, Denicolo and Matteuzzi 2000).
On Cost Functions
For any $n$, and over the domain $x_n \geq 0$, the cost function $C_n(\cdot)$ is convex and nondecreasing on $[0, \infty)$, and continuously differentiable on $(0, \infty)$.

On the Inverse Demand Function
- The inverse demand function $p(\cdot)$ is nonnegative, nonincreasing on $[0, \infty)$, and differentiable on $(0, \infty)$, with $p(0) > \min_n\{C'_n(0)\}$.
- On the interval where $p(\cdot)$ is positive, the function $p(q)q$ is concave in $q$. 
Given a unit tax rate \( t \in (0, p(0) - \min_n \{ C_n'(0) \}) \):

- there exists a Cournot equilibrium \( x \) such that \( X > 0 \).

- there exists a Cournot equilibrium, \( q \), associated with an Ad Valorem tax rate \( \tau^* \), which yields the same aggregate output \( X \), and at least the same tax revenue plus supplier profit as \( x \).

- If \( p'(X) < 0 \), the Cournot equilibrium \( q \) yields a higher tax revenue than \( x \).

- If \( p'(X) < 0 \), and there exist some \( x_i x_j > 0 \) such that \( x_i \neq x_j \), the Cournot equilibrium \( q \) yields higher social welfare than \( x \).
Proof Sketch

Given a unit tax rate $t > 0$ and a Cournot equilibrium $\mathbf{x}$,

- We construct a Cournot equilibrium $\mathbf{q}$ with $\sum_n q_n = \sum_n x_n$, under some ad valorem tax rate $\tau^*$.

- The Cournot equilibrium $\mathbf{q}$ yields at least the same tax revenue as $\mathbf{x}$. If $p'(X) < 0$, it yields a higher tax revenue.

- The equality, $\sum_n q_n = \sum_n x_n$, implies that both Cournot equilibria yield the same aggregate consumer utility, $\int_0^X p(q) dq$.

- Total supplier cost does not increase (strictly decreases) at the Cournot equilibrium $\mathbf{q}$, (if there exist some $x_i x_j > 0$ such that $x_i \neq x_j$).
Proof Sketch: Construction of $q$

If $p'(X) < 0$

Given a unit tax rate $t > 0$ and a Cournot equilibrium $x$,

- We have
  \[
  \begin{cases}
  p(X) - t + x_n p'(X) = C_n'(x_n), & \text{if } x_n > 0, \\
  p(X) - t + x_n p'(X) \leq C_n'(x_n).
  \end{cases}
  \]

- Let $(1 - \tau)p(X) = p(X) - t$. For any supplier $n$ with $x_n > 0$,
  \[
  (1 - \tau)[p(X) + x_n p'(X)] > C_n'(x_n).
  \]

- There exists some $\bar{x}_n > x_n$ such that
  \[
  (1 - \tau)[p(X) + \bar{x}_n p'(X)] = C_n'(\bar{x}_n).
  \]
Proof Sketch: Construction of $q$

If $p'(X) < 0$

There exists some $\bar{x}_n > x_n$ such that

$$(1 - \tau)[p(X) + \bar{x}_n p'(X)] = C'_n(\bar{x}_n).$$

Given $\tau > 0$, we define

$$f_n(\tau) = \begin{cases} 
0, & \text{if } (1 - \tau)p(X) - C'_n(0) \leq 0, \\
x^*, & \text{otherwise},
\end{cases}$$

where $(1 - \tau)[p(X) + x^* p'(X)] = C'_n(x^*).$
Proof Sketch: Construction of $q$

If $p'(X) < 0$

$$fn(\tau) = \begin{cases} 0, & \text{if } (1 - \tau)p(X) - C'_n(0) \leq 0, \\ x^*, & \text{otherwise,} \end{cases}$$

where $(1 - \tau)[p(X) + x^*p'(X)] = C'_n(x^*)$.

- $f_n(\tau)$ is continuous in $\tau$, and is strictly decreasing whenever $f_n(\tau)$ is positive.

- Since $\sum_n f_n(\bar{\tau}) > X$ and $\sum_n f_n(1) = 0$, there exists some $\tau^* \in (\bar{\tau}, 1)$ such that $\sum_n f_n(\tau^*) = X$.

- Let $q_n = f_n(\tau^*)$. $q$ is a Cournot equilibrium with $\sum_n q_n = X$:

$$\begin{cases} (1 - \tau^*)[p(X) + f_n(\tau^*)p'(X)] = C'_n(f_n(\tau^*)), & \text{if } f_n(\tau^*) > 0, \\ (1 - \tau^*)[p(X) + f_n(\tau^*)p'(X)] \leq C'_n(f_n(\tau^*)), & \text{if } f_n(\tau^*) = 0. \end{cases}$$
Given a unit tax rate \( t > 0 \) and a Cournot equilibrium \( \mathbf{x} \),

- We construct a Cournot equilibrium \( \mathbf{q} \) with \( \sum_n q_n = \sum_n x_n \), under some ad valorem tax rate \( \tau^* \).

- The Cournot equilibrium \( \mathbf{q} \) yields at least the same tax revenue as \( \mathbf{x} \). If \( p'(X) < 0 \), it yields a higher tax revenue.

- The equality, \( \sum_n q_n = \sum_n x_n \), implies that both Cournot equilibria yield the same aggregate consumer utility, \( \int_0^X p(q) dq \).

- Total supplier cost does not increase (strictly decreases) at the Cournot equilibrium \( \mathbf{q} \), (if there exist some \( x_i x_j > 0 \) such that \( x_i \neq x_j \)).
Proof Sketch: Supplier Cost Reduction at $q$

**If $p'(X) < 0$**

If $x_i \geq q_i$ and $x_i \geq x_j > 0$, suppose that $x_j < q_j$.

\[-\tau^* p(X) + t \geq -p'(X)(1 - \tau^*)q_j + p'(X)x_j\]
\[> -p'(X)(1 - \tau^*)x_j + p'(X)x_j = p'(X)\tau^* x_j.\]

On the other hand,
\[-\tau^* p(X) + t \leq -p'(X)(1 - \tau^*)q_i + p'(X)x_i\]
\[\leq -p'(X)(1 - \tau^*)x_i + p'(X)x_i = p'(X)\tau^* x_i.\]

**If $p'(X) < 0$**

If $x_i \geq q_i$ and $x_i \geq (>)x_j > 0$, then $x_j \geq (>)q_j$. 

Proof Sketch: Supplier Cost Reduction at $q$

If $p'(X) < 0$

If $0 < x_i \leq q_i$ and $q_i \leq q_j$, suppose that $x_j > q_j$.

\[-\tau^* p(X) + t \geq -p'(X)(1 - \tau^*)q_i + p'(X)x_i \geq -p'(X)(1 - \tau^*)q_i + p'(X)q_i = p'(X)\tau^* q_i.\]

On the other hand,

\[-\tau^* p(X) + t \leq -p'(X)(1 - \tau^*)q_j + p'(X)x_j < -p'(X)(1 - \tau^*)q_j + p'(X)q_j = p'(X)\tau^* q_i.\]

If $p'(X) < 0$

If $x_i \leq q_i$ and $q_i \leq (\times) q_j$, then $x_j \leq (\times) q_j$. 
Proof Sketch: Supplier Cost Reduction at $q$

If $p'(X) < 0$

- If $x_i \geq q_i$ and $x_i \geq (>)x_j > 0$, then $x_j \geq (>)q_j$.
- If $x_i \leq q_i$ and $q_i \leq (<)q_j$, then $x_j \leq (<)q_j$.
- $\sum_n q_n = X = \sum_n x_n$.

\[
\begin{align*}
x_1 & \geq x_2 \geq \cdots \geq x_i \geq \cdots \geq x_N \\
\&\&\&\&
\end{align*}
\]

\[
\begin{align*}
q_1 & \quad q_2 \quad \cdots \quad q_i \quad \cdots \quad q_N \\
\&\&\&\&
\end{align*}
\]

- There exists a threshold $i$ such that $x_j \leq q_j$ for any $j < i$, and $x_j \geq q_j$ for any $j \geq i$.
- For any $m < i \leq n$, $q_m \geq q_n$. 

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Proof Sketch: Supplier Cost Reduction at $q$ 

\[ x_1 \geq x_2 \geq \cdots \geq x_i \geq \cdots \geq x_N \]
\[ q_1 \leq q_2 \leq \cdots \leq q_i \leq \cdots \leq q_N \]

For any $m < i \leq n$, $q_m \geq q_n$.

\[
\begin{align*}
\sum_{j=1}^{i-1} C_j(q_j) - C_j(x_j) & \\
\leq \max_{j=1,\ldots,i-1} C'_j(q_j) \sum_{j=1}^{i-1} q_j - x_j & \\
\leq \min_{j=i,\ldots,N} C'_j(q_j) \sum_{j=1}^{i-1} q_j - x_j & \\
\leq \min_{j=i,\ldots,N} C'_j(x_j) \sum_{j=1}^{i-1} q_j - x_j & \\
\leq \sum_{j=i}^{N} C_j(x_j) - C_j(q_j). & \\
\end{align*}
\]
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4. Efficiency Loss with Taxation
The model

Supplier Profit under Unit Tax
Under a unit (specific) tax rate $t$, supplier $n$’s profit is

$$p(X)x_n - C_n(x_n) - tx_n.$$ 

Supplier Profit under Ad Valorem Tax
Under an ad valorem tax rate $\tau$, supplier $n$’s profit is

$$(1 - \tau)p(X)x_n - C_n(x_n).$$

Affine Inverse Demand Function
$p(q) = b - aq$, for some $a > 0$ and $b > 0$. 

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Efficiency Loss

Definition

The **efficiency** of a nonnegative vector \( x = (x_1, \ldots, x_N) \) is defined as

\[
\gamma(x) = \frac{\int_0^{\sum_{n=1}^N x_n} (b - aq) dq - \sum_{n=1}^N C(x_n)}{\int_0^{\sum_{n=1}^N x_n^S} (b - aq) dq - \sum_{n=1}^N C(x_n^S)},
\]

where \( (x_1^S, \ldots, x_N^S) \) is an optimal solution to the following optimization problem,

\[
\text{maximize} \quad \int_0^{\sum_{n=1}^N x_n} (b - aq) dq - \sum_{n=1}^N C_n(x_n)
\]

subject to \( x_n \geq 0, \quad n = 1, 2, \ldots, N. \)
Efficiency Loss under Unit Tax

Assumptions

- For any $n$, the cost function $C_n(\cdot)$ is convex and nondecreasing on $[0, \infty)$, and continuously differentiable on $(0, \infty)$, with $C_n(0) = 0$.
- The price at zero supply is larger than the minimum marginal cost, i.e., $b > \min_n\{C'_n(0)\}$.

Theorem

Given a unit tax rate $t \in [0, b]$, we have the following tight efficiency lower bound for any Cournot equilibrium $x$:

$$\gamma(x) \geq h(t/b) = \begin{cases} 
- \left( \frac{t}{b} \right)^2 + \frac{2}{3}, & \text{if } \frac{t}{b} \in [0, 1/3], \\
- \frac{1}{4} \left( \frac{t}{b} \right)^2 - \frac{t}{2b} + \frac{3}{4}, & \text{if } \frac{t}{b} \in (1/3, 1].
\end{cases}$$
Efficiency Lower Bound under Unit Tax

\[ h(t/b) \]

\[(0, 2/3)\]
Assumptions

- For any $n$, the cost function $C_n(\cdot)$ is convex and nondecreasing on $[0, \infty)$, and continuously differentiable on $(0, \infty)$, with $C_n(0) = 0$.
- The price at zero supply is larger than the minimum marginal cost, i.e., $b > \min_n \{C'_n(0)\}$.

Theorem

Given an ad valorem tax rate $\tau \in [0, 1]$, we have the following tight efficiency lower bound for any Cournot equilibrium $\mathbf{x}$:

$$
\gamma(\mathbf{x}) \geq g(\tau) = \begin{cases} 
-\tau^2 - 2\tau + 2, & \text{if } \tau \in [0, 1/2], \\
3 - 4\tau, & \text{if } \tau \in (1/2, 1), \\
3/4, & \text{if } \tau = 1.
\end{cases}
$$
Efficiency Lower Bound under Ad Valorem Tax

\[ g(\tau) \]

\( (0, \frac{2}{3}) \) and \( (1, 0) \) points on the graph.
Efficiency Loss in Cournot Oligopolies

- When the inverse demand function is convex, we establish a lower bound on the efficiency of Cournot equilibria in terms of a scalar parameter derived from the inverse demand function.

- We show that Ad Valorem tax welfare dominates unit tax, for a Cournot oligopoly model with general convex cost functions.

- For affine inverse demand functions, we establish tight efficiency lower bounds under unit and ad valorem taxation.