Dynamic Traffic Congestion Pricing Mechanism with User-Centric Considerations

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Abstract—We consider the problem of designing real-time traffic routing systems in urban areas. Optimal dynamic routing for multiple passengers is known to be computationally hard due to its combinatorial nature. To overcome this difficulty, we propose a novel mechanism called User-Centric Dynamic Pricing (UCDP) based on recent advances in algorithmic mechanism design. The mechanism allows for congestion-free traffic in general road networks with heterogeneous users, while satisfying each user’s travel preference. The mechanism first informs whether a passenger should use public transportation or the road network. In the latter case, a passenger reports his maximum accepted travel time with a lower bound announced publicly by the road authority. The mechanism then assigns the passenger a path that matches with his preference given the current traffic condition in the network. The proposed mechanism introduces a fairness constrained shortest path (FCSP) problem with a special structure, thus enabling polynomial time computation of path allocation that maximizes the sequential social surplus and guarantees fairness among passengers. The tolls of paths are then computed according to marginal cost payments. We show that reporting true preference is a weakly dominant strategy. The superior performance of the proposed mechanism is demonstrated on several simulated routing experiments in comparison to user equilibrium and system optimum.

I. INTRODUCTION

Intelligent and scalable traffic routing for large cities is an active research area in recent years. Many route guidance systems have been used to assist drivers in path choice decision making by simply computing the shortest path from a source to a destination regardless of the changing conditions of the roadway [1]. More advanced systems are proposed to assign users concrete routes after performing some computations from a macroscopic point of view [2]. In these systems, the authors consider static flows of multiple homogeneous users on the networks and attempt to compute an a priori distribution of the users on the road networks. This static traffic assignment (STA) problem can be solved by using the Wardrop principle [3] or Karush-Kuhn-Tucker conditions to achieve user equilibrium (UE) solution or social optimal (SO) solution respectively. At the UE state, each user selects his fastest route, while at the SO state, the total travel time of all drivers is minimized. Although SO is more preferable for road authorities, it encounters an unfairness issue as some users are assigned longer paths to allow the system optimum. Thus, in [8] and [9], the authors propose the constrained system optimum (CSO) to capture the above two aspects. In their CSO models, they consider a traffic assignment pattern for homogeneous users that minimizes the total travel time subject to a fairness constraint via a single tolerance factor. However, this pre-specified tolerance factor does not truly reflect fairness for individual traveler whose preference and purpose are unrevealed. In addition, congestion can occur when the tolerance factor is not properly chosen even if the road network is capable of supporting congestion-free traffic at the SO state. Furthermore, the authors do not consider dynamic traffic flows and the selfish behavior of rational passengers. Moreover, a rational passenger would not use the routing system and follow its recommendation if such compliance does not provide him with a tangible gain. Apparently, these limitations have prevented a direct application of the model in real-world situations.

In fact, to partially restrain the selfish behavior of travelers that can lead to traffic congestion during rush hours, tolling systems such as Electronic Road Pricing (ERP) in Singapore are deployed [10]. The positive effects of the ERP system on spreading peak-hour travel demand are verified in [11] and [12]. At the time of this writing, pre-computed time-varying tolls within a day are used to affect passengers’ behavior. However, this time-varying tolling scheme does not enable congestion-free traffic flows because urban traffic also varies from day to day. A similar variable price scheme based on the evolutionary approach is proposed in [13] to enable congestion-free traffic when travel demand is static. In this scheme, passengers will decide to travel or stay at home permanently after a long adjustment process in response to variable tolls set by the road authority. Not only is the convergence process slow when the demand is static, but the traffic flow is also unstable when the demand is dynamic. As an attempt to compute flow-dependent tolls in dynamic environment, the work presented in [14] proposes a toll design method that combines road pricing with DTA. The authors formulate the problem as a bi-level programing problem, in which the road pricing model represents the upper-level decision making process, and the DTA model represents the lower-level counterpart. The resulting formulation is an NP-hard problem [14]. One possible method to obtain a locally

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optimal solution suggested by the author is to combine grid search for the upper-level optimization and simulation for the DTA in an iterative procedure. However, such a method scales poorly with the size of the network and does not provide insightful understanding of the properties of obtained solutions for further improvement. We emphasize that all the mentioned pricing schemes have not addressed the unfairness issue among individuals.

Therefore, in recent years, many researchers have adopted a game-theoretic approach to handle the above difficulties from the microscopic perspective [15], [16]. Recent advances in algorithmic mechanism design [17] provides a promising approach to incentivize rational participants, or agents, to cooperate with the system in order to reach desirable outcomes. This approach motivates agents to disclose their private information. As usually done in market mechanism design [18], designers aim to construct a mechanism that has individual rationality (IR) and incentive compatibility (IC) properties. The former means agents do not suffer any loss when they use the system, and the latter means revealing truthful information is in their best interest. In algorithmic mechanism design, besides the IR and IC properties, designers also concern about computational complexity when computing an allocation rule and a payment rule for intended outcomes [19]. The key technical difficulties lie in the combinatorial nature of the allocation rule and the interweaving relationship of allocation rules and payment rules.

In our case, passengers can be viewed as agents in a routing game created by the interaction among passengers and the road authority. Mechanism design for routing games have been studied extensively in computer networks [20]–[22] while very few works have been done for urban transportation networks [23]. Although both types of networks share some similar modeling aspects, inherent characteristics of the two types of networks differ substantially. In particular, in computer networks, as discussed in [24], utility functions of users are unknown to the network; and even if the utility functions are known, there is no central authority that knows all the link capacities and network topology. In addition, autonomous systems (AS) in the network can run any routing algorithm that benefits them the most. Therefore, we can hardly simulate the flows for the entire computer networks such as the Internet. In contrast, in transportation networks, passenger utility functions can be modeled quite accurately based on regression analysis [25]. In addition, we can monitor real-time traffic flows via sensor networks [26]. Thus, current models and results in computer network routing cannot be used and applied in urban transportation network routing.

The most related work in mechanism design for urban transportation routing is [23]. In their work, the authors propose a day-to-day auction mechanism in which a road user has to bid everyday to purchase a bundle of network permits that allows him to use his preferred path. Although this approach can reduce the computational complexity of finding an allocation rule for a single OD pair network, it is not clear how to determine the allocation rule for general network with multiple OD pairs. The day-to-day auction mechanism is also not practical to be implemented for real-world road networks due to several reasons. The mechanism requires users to go through a bidding process before being able to use the paths, while in reality, road users often need an instantaneous route choice decision. Furthermore, although the authors show that truthfully reporting the valuation of several bundles of permits is a dominant strategy for each user, it is difficult for users to determine their true valuation.

To overcome the computational complexity and satisfy game-theoretic requirements of transportation network routing problem, in this work, we propose a novel mechanism called User-Centric Dynamic Pricing (UCDP). The mechanism does not only allow for congestion-free traffic in general road networks with dynamic flows of heterogeneous users but also satisfies each user’s travel preference. The mechanism works as follows. When a new passenger arrives at an origin node, he receives information from the authority on the current network condition. The authority either suggests using public transportation if the network is about to be congested or provides him with the minimum travel time required to complete his trip. In the latter case, the passenger then reports his maximum tolerated travel time that is not less than the minimum travel time announced by the authority. Based on the current traffic condition in the network, the mechanism then offers the passenger a path to his destination that matches with his preference. The UCDP mechanism takes into account the fairness concern when allocating paths to travelers. The tolls of paths are computed according to the marginal cost pricing principle. This mechanism is applicable for mobility-on-demand systems [27] where fleets of autonomous cars [28] are used in future cities to pick up and deliver passengers.

The main contribution of this work is a novel mechanism that has two significant properties. First, the mechanism is user-centric in the sense that it considers both passengers’ preference and fairness among individuals. At the same time, the mechanism can achieve maximum sequential social surplus, i.e. the mechanism is efficient. We show that reporting true preference is a weakly dominant strategy. Second, the mechanism introduces a new fairness constrained shortest path (FCSP) problem with a special structure, thus enabling polynomial time computation of path allocation. Therefore, we can handle general networks having multiple OD pairs with dynamic flows of heterogeneous users in a computationally efficient way.

This paper is organized as follows. In Section II, a formal problem statement is given. The allocation rule and the payment rule are described and analyzed in Section III. In Section IV, we provide algorithms to compute dynamic price in real-time. The superior performance of the proposed mechanism is demonstrated on several simulated routing experiments in Section V. We conclude the paper with final remarks in Section VI.

II. PROBLEM STATEMENT

We first present the dynamic traffic network model and then discuss how the UCDP mechanism works.

A. Dynamic Network Model

We consider a general traffic network $G = (V, E)$ with nodes $V$ and directed links $E$. We assume that there are $L$ demand traffic flows in the network, denoted by $\{d_l\}_{l=1}^L$. Each demand flow $d_l$ can be represented as a renewal process with rate $\lambda_l > 0$ arriving at nodes $R$, where $R \subseteq V$.

We assume that each vehicle has only one passenger. Each
passenger from these $L$ demand flows has a request to travel from an origin $r \in V$ to a destination $s \in V$. Between each origin-destination (OD) pair $(r, s)$, there exists path $p \in P^{rs}$, where $P^{rs}$ is the set of all feasible paths between $r$ and $s$. We define $\mathcal{P}$ as the set of all $P^{rs}$. In addition, for each OD pair $(r, s)$, we assume that there is public transportation such as train or light rail that connects $r$ and $s$ with infinite capacity.

We define $x = (k, q) \in X$ as the state of the network at any time instant, where $k$ is density, measured by the number of vehicles per kilometer and $q$ is the number of travelers on all links $E$. In particular, we have $k = \sum_{e \in E} k_e$ and $q = \sum_{e \in E} q_e$ in which $k_e$ and $q_e$ denote the density and the number of travelers on a particular link $e \in E$ at that time instant. Each link $e \in E$ has a jam density $K_{je}$ at which the traffic on the link is congested with zero traffic flow. Based on the Greenshields model [29], $K_{ce} = \frac{1}{2}K_{je}$ denotes a critical density at which additional input of vehicles on the link decreases traffic flow and eventually leads to traffic congestion. The set of all feasible density on link $e \in E$ is $K_e$. We define $K = \prod_{e \in E} K_e$ so that $k \in K$. Similarly, the set of all feasible number of people on link $e$ is $Q_e$, and $q \in Q = \prod_{e \in E} Q_e$. We note that $x$ is time-varying.

At any time instant $t$, assume that the network state is $x$, we would like to simulate the network state $\tilde{x} \in X$ after a path $p \in \mathcal{P}$ is assigned to a new traveler given that all current travelers in the network are following their assigned paths. Ideally, we would like to predict $\tilde{x}$ dynamically at any time instant in the future after the path $p$ is assigned to the new passenger. However, to simplify the notations in the following discussion, we assume that from that time instant $t$, the state $x$ of the network will not change until the new passenger completes his trip. We can simulate $\tilde{x}$ by simply adding the new traveler on each link $e \in p \subset E$ as if he were traveling on $e$ at time $t$. Thereafter, given density on link $e$, the space-average speed of passengers traveling on the link can be computed by using the Greenshields model [29]. In particular, $\tilde{x} = G(x, p)$ where $G : X \times \mathcal{P} \to X$. We will return to dynamic and real-time prediction in Section IV.

The travel time on each link $e$, denoted by $\tau_e(\tilde{k}_e)$, is a strictly increasing and convex function of density $\tilde{k}_e$, where $\tilde{k}_e$ is extracted from $\tilde{x} = (\tilde{k}, \tilde{q}) = G(x, p)$ and is the experienced density of the driver. For a new driver who follows a path $p \in \mathcal{P}$, his experienced travel time along this path would be:

$$
\tau(x, p) = \sum_{e \in p} \tau_e(\tilde{k}_e).
$$

### B. Dynamic Pricing Mechanism

We now discuss the UCDP mechanism in detail. Let $H = \{H_n\}_{n=1}^\infty$ be the set of all heterogeneous passengers where $H_n$ denotes a class of homogeneous passengers who have the same value of time $\tau_n$. If passenger $i$ belongs to class $H_n$, we know that his value of time is $\alpha_i = \tau_n$. Here we assume that we can classify a passenger based on some demographic data such as tax and income. At time $t$, a passenger $i \in H_n$ who is the latest incoming passenger from $L$ demand flows wants to travel from $r \in V$ to $s \in V$. Passenger $i$ will report his maximum accepted travel time $\theta_i \geq \theta_{\text{min}}$, measured in time units, to a central system. In game-theory terminology, we define $\theta_i$ as the type of passenger $i$. The lower bound $\theta_{\text{min}}$ is the minimum time to travel from $r$ to $s$ and is announced publicly by the road authority. The value of $\theta_{\text{max}}$ is computed in Section IV. If there is currently no path to travel from $r$ to $s$ due to congestion, we define $\theta_{\text{max}} = +\infty$, and the passenger is advised to use public transportation. Otherwise, our mechanism then computes a suitable path $p$ for this passenger and a toll $w_i(x, p)$ that he has to pay to complete his trip. The utility function of passenger $i$ for such an assignment is determined as follows:

$$
u_i(x, p, \theta_i) = \alpha_i(\theta_i - \tau(x, p)) - w_i(x, p),$$

where $\tau(x, p)$ is the travel time that passenger $i$ spends when traveling along path $p$ from $r$ to $s$ and can be computed by Eq. 1. We assume that passenger $i$ is risk-neutral and seeks to maximize his utility $\nu_i(x, p, \theta_i)$ of the trip. Clearly, the first term in Eq. 2 can be considered as passenger $i$’s benefit of making the trip, and the second term is his cost. Thus, we denote $\nu_i(x, p, \theta_i) = \alpha_i(\theta_i - \tau(x, p))$ as passenger $i$’s benefit.

The road authority, on the other hand, tries to maximize the social surplus of all current travelers in the road network. When a path $p$ is assigned to passenger $i$, the road authority’s utility function is defined as follows:

$$V(x, p) = -\sum_{e \in E} \alpha_0^e \tau_e(\tilde{k}_e) q_e,$$

where $\alpha_0^e$ is the authority’s value of time for passenger $i$ on link $e$, which depends on this passenger’s class, and $\tilde{k}_e$, extracted from $\tilde{x} = G(x, p)$, is the new density on link $e$ when passenger $i$ travels along path $p$. In particular, if $e \notin p$, we have $\tilde{k}_e = k_e$, and if $e \in p$, $\tilde{k}_e$ is the induced density due to $q_e + 1$ travelers on link $e$. Equation 3 implies that the road authority is better off if the total travel time of current travelers is smaller.

As we are handling dynamic traffic network in which demands arrive as renewal processes, the ordinary social optimal concept is not suitable. Thus, we propose sequential social surplus as an alternative measurement of total society’s benefit. More precisely, sequential social surplus is the sequence of the sum of road authority utility $V(x, p)$ and a new traveler’s benefit $\nu_i(x, p, \theta_i)$ over time.

Our goal is to assign each passenger $i$ a path $p$ that satisfies his travel time constraint $\theta_i$ while maximizing sequential social surplus when passenger $i$ joins the network. In addition, we want to ensure that the density on each link $e \in E$ does not exceed its critical density $K_{ce}$ for all times $t$, i.e. the network is always congestion-free. The exceeding demand is rerouted to public transportation. While we assume that each passenger will follow the assigned path, there is a concern that a passenger $i$ could increase his utility $\nu_i(x, p, \theta_i)$ if he lies about his true tolerated travel time $\theta_i$. Such manipulation would negatively affect the overall network efficiency. To avoid this situation, our approach, described in Sections III and IV, computes a path allocation rule and a payment rule that are incentive compatible (or strategy-proof), which means passengers have no incentive to lie about their preference.

### III. APPROACH

#### A. Allocation Rule

Recall that our first objective is to achieve a path allocation rule that matches with each user’s preference while maximizing sequential social surplus and avoiding congestion. More
precisely, given that at time $t$, a passenger $i$ reports his type $\theta'_i$, we aim to solve the following optimization problem for all $\theta'_i$:

$$\max_{p \in P_\theta} V(x, p) + v_i(x, p, \theta'_i)$$

subject to

$$\tau(x, p) \leq \theta'_i,$$
$$\hat{k}_e \leq K_{C_e}, \ \forall e \in E.$$ 

The objective function can be rewritten as follows:

$$V(x, p) + v_i(x, p, \theta'_i) = -\sum_{e \in E} \alpha_0^i \tau_e(\hat{k}_e) q_e + \alpha_i (\theta'_i - \tau(x, p))$$

$$= -\sum_{e \in E} \alpha_0^i \tau_e(\hat{k}_e) q_e + \alpha_i \theta'_i - \sum_{e \in p} \tau_e(\hat{k}_e)(\alpha_0^i q_e + \alpha_i)$$

$$= -\sum_{e \in E} \alpha_0^i \tau_e(\hat{k}_e) q_e + \alpha_i \theta'_i$$

$$- \sum_{e \in p} \left\{ \tau_e(\hat{k}_e) - \tau_e(\hat{k}_e) \right\} \alpha_0^i q_e + \tau_e(\hat{k}_e) \alpha_i \right\},$$

where $q_e$ is the number of travelers in each link right before passenger $i$ starts his trip.

Since we know the network state $x$ and passenger $i$’s reported type as well as his value of time, the terms $\sum_{e \in E} \alpha_0^i \tau_e(\hat{k}_e) q_e$ and $\alpha_i \theta'_i$ are constant. Therefore, our problem becomes a fairness constrained shortest path (FCSP) with bounded travel time:

$$\min_{p \in P_\theta} \sum_{e \in p} \left\{ \tau_e(\hat{k}_e) - \tau_e(\hat{k}_e) \right\} \alpha_0^i q_e + \tau_e(\hat{k}_e) \alpha_i \right\}$$

subject to

$$\tau(x, p) \leq \theta'_i,$$
$$\hat{k}_e \leq K_{C_e}, \ \forall e \in E.$$ 

The equivalent arc-based formulation is as follows:

$$\min \sum_{e \in E} z_e \left\{ \tau_e(\hat{k}_e) - \tau_e(\hat{k}_e) \right\} \alpha_0^i q_e + \tau_e(\hat{k}_e) \alpha_i \right\}$$

Subject to

$$\sum_{e \in p^s} z_e = 1, \ \sum_{e \in p^s} -z_e = -1,$$ 

$$\sum_{e \in v^+} z_e - \sum_{e \in v^-} z_e = 0, \ \forall v \in V \backslash \{r, s\},$$ 

$$\sum_{e \in E} z_e \left[ \tau_e(\hat{k}_e) \right] \leq \theta'_i,$$
$$\hat{k}_e \leq K_{C_e}, \ \forall e \in E,$$ 

$$z_e \in \{0, 1\}, \ \forall e \in E.$$

The objective function in Eq. 7 is minimizing the sum of the increment in total generalized travel time of current passengers on the roads and the generalized travel time of the new passenger. Here $z_e$ is a binary variable, i.e., it is 1 if link $e$ is on path $p$ assigned for passenger $i$, and zero otherwise. The coefficients of $z_e$ in the objective function are called edge costs. The first two constraints in Eq. 8 are the condition that passenger $i$ needs to make a trip from $r$ to $s$, where $r^+$ is the set of links going out from origin $r$ and $s^-$ is the set of links coming in destination $s$. Eq. 9 is a node balance constraint equation, where $v^+$ represents the set of links coming in node $v \in V \backslash \{r, s\}$ and $v^-$ represents the set of links going out from $v$. The constraint in Eq. 10 expresses the important requirement that the travel time of passenger $i$ along his assigned path should not exceed his stated maximum accepted travel time. The coefficients of $z_e$ in this constraint are called edge weights. The constraint in Eq. 11 restricts on link density to guarantee that congestion will not happen. We note that, to further utilize the roads, the upper bound $K_{C_e}$ in Eq. 11 can be set to a higher value without changing the algorithm in Section IV. If the preference constraint in Eq. 10 and the link density constraint in Eq. 11 are omitted, an optimal solution to this formulation would coincide with the social optimum in traditional traffic assignment at this time instant. In addition, it is worth noting that our objective function also demonstrates how the value of time of passenger $i$ can affect his assigned path. As the ratio of $\alpha_i$ to $\alpha_0^i q_e$ becomes extremely large, i.e., passenger $i$ needs to reach the destination urgently, the road authority can deliberately neglect his value of time and can only consider to minimize travel time on each link for this passenger. Hence, the formulation has significant meaning in real-world routing when we need to provide a police car or an ambulance with the fastest path to reach the accident site. Therefore, the combination of the preference constraint and the role of passenger’s value of time in the objective function successfully model fairness requirement for individuals. To the best of our knowledge, the proposed model is the first in the literature that considers all of the above aspects.

Regarding computational complexity of the problem, it is well known that the general shortest path problems with additional constraints and possible negative edge costs are NP-hard [30]. However, by leveraging the properties of transportation networks and the novel design of our mechanism, we can naturally handle the problem by assuming that travel times on all roads in the network are positive and uniformly bounded away from zero. Under these mild assumptions, our FCSP problem can be solved in polynomial time by using the label-correcting algorithm [31], [32]. The implementation of label-correcting algorithm will be explained in detail in Section IV.

B. Payment Rule

We now discuss how to design the payment rule such that the proposed mechanism satisfies incentive compatibility. Given that we have solved the FCSP problem in Eq. 4-6, with $\theta'_i$, we obtain an optimal solution $p^*(\theta'_i)$, which is the path that we will allocate to passenger $i$. With this optimal path, we can calculate the payment that passenger $i$ pays for using the path based on marginal cost pricing principle:

$$w_i(x, p^*(\theta'_i)) = V(x, \emptyset) - V(x, p^*(\theta'_i))$$

$$= \sum_{e \in P^*(\theta'_i)} \left[ \tau_e(\hat{k}_e) - \tau_e(\hat{k}_e) \right] \alpha_0^i q_e.$$ 

The first term on the right side of Eq. 13 is the maximum value of the authority’s utility when passenger $i$ is not included, and the second term is the authority’s utility when
passenger $i$ travels along path $p^*(\theta'_i)$. We note that tolls for different classes of passengers are different, depending on the authority’s value of time $\alpha_{0,\text{e}}$ for a particular class.

This payment rule would favor low-income groups and thus provides fairness among individuals as discussed by [11]. Under this price, the utility of passenger $i$, $u_i(x, p^*(\theta'_i), \theta_i)$, in Eq. 2 becomes:

$$\alpha_i(\theta_i - \tau(x, p^*(\theta'_i)) - V(x, \emptyset) + V(x, p^*(\theta'_i)).$$ (14)

The pricing mechanism we propose looks similar to the celebrated Vickrey-Clarke-Groves (VCG) mechanism that is incentive compatible [33], [34]. Nevertheless, our mechanism is considerably different from VCG mechanism because at each “game”, we only consider a single player, i.e., a passenger, who indirectly plays with all current passengers in the network before actually joining the network. Moreover, we have additional preference and capacity constraints in our allocation rule problem, while VCG mechanism does not have. Therefore, it is necessary to prove that our mechanism still maintains strategy-proof property despite of the modifications as follows.

**Theorem 1** Given the allocation rule determined by the FCSP problem and the marginal cost payment rule as in Eq. 13, reporting $\theta'_i = \theta_i$ is a weakly dominant strategy of passenger $i$, $\forall i \in H$.

**Proof:** Assume that truth telling is not a weakly dominant strategy for some $i$, i.e. there exists some $\theta_i$ and $\theta'_i$ such that

$$u_i(x, p^*(\theta'_i), \theta_i) > u_i(x, p^*(\theta_i), \theta_i),$$ (15)

$$\tau(x, p^*(\theta'_i)) \leq \theta_i,$$ (16)

and the network is congestion-free under both reports $\theta_i$ and $\theta'_i$. Equation 16 means that $p^*(\theta'_i)$ is a feasible solution to the optimization problem:

$$\max_{p \in \mathcal{P}_x} V(x, p) + v_i(x, p, \theta_i)$$

subject to

$$\tau(x, p) \leq \theta_i,$$

$$\hat{c}_e(x) \leq \hat{K}_e, \quad \forall e \in E.$$ (17)

We have $p^*(\theta_i)$ as the optimal solution to the above problem due to the allocation rule. Thus:

$$V(x, p^*(\theta'_i)) + v_i(x, p^*(\theta'_i), \theta_i) \leq V(x, p^*(\theta_i)) + v_i(x, p^*(\theta_i), \theta_i).$$

(17)

On the other hand, substituting Eq. 14 into Eq. 15, this yields

$$\alpha_i(\theta_i - \tau(x, p^*(\theta'_i)) - V(x, \emptyset) + V(x, p^*(\theta'_i))$$

$$> \alpha_i(\theta_i - \tau(x, p^*(\theta_i)) - V(x, \emptyset) + V(x, p^*(\theta_i))$$

$$\Leftrightarrow V(x, p^*(\theta'_i)) + v_i(x, p^*(\theta'_i), \theta_i)$$

$$> V(x, p^*(\theta_i)) + v_i(x, p^*(\theta_i), \theta_i).$$

Comparing the above inequality with Eq. 17, we reach a contradiction.

**IV. UCDP Computation**

In this section, we describe in detail how we allocate paths and compute tolls dynamically. We first show how traffic flows are simulated in the UCDP algorithm and then provide two algorithms for computing minimum travel times, dynamic path allocation and dynamic tolls.

**A. Traffic simulation**

Assuming that travelers in the networks are following assigned paths, we use the Greenshields model to simulate traffic state in the network at any time instant. The density $k_e$ on each link $e$ can be computed by counting the number of passengers on that link. Given maximum speed $v_{\text{e, max}}$ and jam density $K_{Je}$ on each link $e \in E$, the space-average speed $v_e$ and travel time $\tau_e(\hat{k}_e)$ on each link $e$ at each time instant are computed as: $v_e = v_{\text{e, max}} \left(1 - \frac{\hat{k}_e}{K_{Je}}\right)$, and $\tau_e(\hat{k}_e) = \frac{\hat{k}_e}{V_e}$, where $L_e$ is the physical length of link $e$. The traffic flow and the maximum traffic flow on each link $e$ at each time instant are $f_e = v_e \hat{k}_e$ and $F_{\text{e, max}} = \frac{1}{V_e} v_{\text{e, max}} K_{Je}$ respectively.

The above computation process can also be used to predict future network states as passengers in the network are following assigned paths. Hence, we assume that there exists a procedure $\text{Greenshields}(x, t)$ that predicts the future network state from the current network state $x$ after a period of time $t$. Thus, the more accurate prediction of $\hat{x} = \hat{G}(x, p)$ in Sections II-III can be done by calling the procedure $\text{Greenshields}$ before accessing an edge state (see Algorithm 1). The computed states are real-time and dynamic.

**B. Minimum travel time computation**

We compute minimum travel times using Dijkstra’s algorithm on the graph with time-varying edge costs. The edge cost for link $e \in E$ is the travel time $\tau_e(\hat{k}_e)$ where $\hat{k}_e$ is computed at the time the passenger reach link $e$ from the current network state $x$ by calling the procedure $\text{Greenshields}$. In addition, in this algorithm, a link with density greater than the upper bound in Eq. 11 is considered to be unavailable.

**C. Dynamic allocation and pricing algorithm**

Algorithm 1 shows how assigned paths and tolls are computed dynamically from an origin node $r \in V$ to a destination node $s \in V$. We use a label-correcting algorithm to find all Pareto-optimal paths. The details of the algorithm can be found in [31], [32]. Essentially, at each vertex, we maintain a set of labels ($C, A$), each of them consisting of a cost component $C$ and a weight component $A$. In particular, at vertex $v \in V$, the cost component of a label is the cost-of-arriving from $r$ to $v$ as defined in the FCSP problem (see Section III) by following the path induced by the label. Similarly, the weight component is the total travel time from $r$ to $v$. At Line 8, we compute the edge costs, denoted as $\hat{c}_e$, and the edge weights, denoted as $\hat{a}_e$, from the FCSP problem after predicting the future network states at Line 7. The priority queue $Q$ (Line 2) stores labels based on the cost component. Finally, the path with smallest cost from $r$ to $s$ and the associated toll is computed at Lines 23-24. We note that as the algorithm computes all Pareto-optimal paths, we can return a list of paths and corresponding tolls for users to choose from. Nevertheless, based on our model, a rational
user would choose the path with smallest cost \( C_s \) as currently returned in Algorithm 1 since this path maximizes the user’s utility. The complexity of this algorithm is \( O(\theta'^2 |V|^2) \) where \( \theta' \) is the reported maximum tolerated time and \( |V| \) is the number of nodes [31], [32].

V. EXPERIMENTS

In the first experiment, we tested the UCDP mechanism on a parallel-link network with one OD pair as shown in Fig. 1(a). The specification for this network (length, maximum speed, critical density, and maximum flow for all links) is shown in Table I. The demand flow rate, \( \lambda_1 \), is 970 passengers per hour. Passengers can also use public transportation to reach the destination in one hour. The value of times for all passengers are 10 USD/h, and the value of time of the authority, \( \alpha_n^{10} \), is 100 USD/h for all links.

We compared the performance of the UCDP mechanism with user-equilibrium (UE) and social-optimal (SO) performance, and the results are presented in Fig. 2. Since we are dealing with dynamic demand flows, we can not compute a priori UE and SO performance as usually done in static traffic assignment. Thus, UE and SO performance in Fig. 2 is computed sequentially for every new arriving passenger. As shown in Figs. 2(a)-2(c), when all passengers act selfishly to choose paths with the minimum traveling times at the time they arrive at the origin vertex, the densities, link travel times, and flows for all links reach their critical values after about 2.2 hours. Therefore, under the UE behavior of passengers, the road network reaches the congestion condition rapidly. Subsequently, Fig. 2(d) shows that a large number of passengers travel by public transportation while others who use the road network spend about 0.82 hours to reach their destination.

In contrast, the UCDP mechanism is able to maintain congestion-free traffic flows on all links. As we can see in Fig. 2(e), the densities on three links are always smaller than the corresponding critical densities, which means congestion does not happen on these links. This observation is the direct consequence of the constraints considered in the FCSP in Section III. Therefore, the link travel times in Fig. 2(f) are stabilized, and the traffic flows in Fig. 2(g) are close to the corresponding maximum flows. We report the travel time for each passenger under the UCDP mechanism in Fig. 2(h). At equilibrium, we expect that all passengers will report their true maximum tolerated travel time due to the strategy-proof property of the mechanism. The plot indicates that all passengers’ preferences are satisfied, and their allocated travel times are generally different from the announced minimum times. In addition, the travel time under the UCDP mechanism in Fig. 2(h) is about 0.14 hours, which is about 5.7 times faster than the UE performance in Fig. 2(d). We recall that the mechanism use tolls to incentivize passengers to reveal their true maximum tolerated travel time. The tolls, generalized costs (negation of utilities), and benefits for passengers are thus shown in Fig. 2(i). We emphasize that, at equilibrium, all passengers’ generalized cost are minimized.

To compare the performance of the UCDP mechanism with SO performance, we plot the ratio of the total travel time of all passengers in the network when UCDP is used to the total travel time of all passengers when they are coordinated in a socially optimal way in Fig. 2(j). The value of UCDP total travel time and the difference between total travel time in UCDP and SO cases are depicted in Fig. 2(k). The two plots indicate that UCDP performance is very close to the SO performance in this experiment. We note that as total demand (\( \lambda_1 = 970 \)) is less than total maximum traffic
flows (1187.5) on the three routes, at SO state, there is no congestion. Thus, we emphasize that in UCDP mechanism, we have achieved near SO performance without congestion and at the same time satisfied all passengers’ preferences. This observation highlights the game-theoretic aspect of the UCDP mechanism and how the mechanism allocates near SO paths at equilibrium.

In the second experiment, we demonstrated how the road authority uses the UCDP mechanism to control the traffic flows when road conditions change over time. We consider a general road network as shown in Fig. 1(b) with the specification in Table II. From the table, link 23 has the normal maximum speed 120 km/h, but when the link is disrupted, the maximum speed is 20 km/h. There are two
demand flows from vertex 0 to vertex 1 and from vertex 4 to vertex 5, each with rate 400 passengers per hour. We assume that value of time for passengers is 10 USD/h. In normal operation, value of time of the road authority is 100 USD/h, but when the link 23 is disrupted, the road authority’s value of time for link 23 is 400 USD/h.

In Fig. 3, we show the road network condition over time when the disruption occurs. When passengers are rerouted due to the disruption, passengers are assigned longer paths with lower tolls, and a majority of passengers is quickly restored to the normal operation under the UCDP mechanism. Finally, we show how tolls vary through this disruption in Fig. 3(d). As we can see, a majority of passengers is assigned longer paths with lower tolls, and a small fraction of passengers use link 23 in their allocated paths with higher tolls. This result reflects the effect of the authority’s value of time for link 23 under disruption on the distribution of passengers in the road network.

VI. CONCLUSIONS

We have introduced and analyzed the User-Centric Dynamic Pricing (UCDP) mechanism that is promisingly implementable for urban transportation networks. The mechanism is efficient and explicitly focuses on users’ travel preference and fairness among users. Therefore, the resulting routing algorithm is pleasant and easy to use from passengers’ perspective. Passengers are always better off to report their true traveling time tolerance. Furthermore, viewed from a broader perspective, this novel mechanism equips governments with instruments to achieve sustainable transportation systems by reducing urban congestion and addressing related social and environmental impacts. The enabling technical idea lies in the new formulation of the fairness constrained shortest path (FCSP) problem that can be solved in polynomial time. Our analysis and experimental results justify the strategy-proof property of the mechanism and its computational feasibility.

Future extension of the work is broad. We will incorporate the individual rationality aspect in our work so that passengers will naturally join our system. We also plan to provide the distributed implementation of the mechanism in our future work.

REFERENCES