Magnet Arrays for Synchronous Machines

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Abstract

The Halbach array was developed for use as an optical element in particle accelerators. These arrays appear to have utility as the permanent magnet structure in synchronous machines. Useful geometries of such arrays are presented in Cartesian, polar, and cylindrical coordinates. The design of a novel linear motor incorporating a Halbach array is given. This motor has been optimized for use in a magnetic suspension stage for photolithography; the paper presents the details of the motor design and its force and power characteristics.

1 Introduction

With the advent of rare earth magnetic materials which combine high remanence and coercive force, new application areas have opened for synchronous permanent magnet machines. Typically, such machines use periodic permanent magnet structures wherein the magnetization vector is directed purely normal to the machine air gap. The magnitude of this magnetization alternates periodically as a function of the dimension parallel to the machine air gap.

An interesting alternative to this conventional solution, which has not yet been widely used in synchronous machines, is the magnet array geometry introduced by Halbach for use in particle accelerator optical elements and advanced synchrotron light sources. In this geometry, the magnetization vector in the array has both gap-normal and gap-tangential periodic components.

The purpose of this paper is to demonstrate that the Halbach array is highly applicable for magnetic excitation in synchronous machines. To this end, we present the geometry of Halbach arrays in Cartesian, polar, and cylindrical coordinates. We also present the design of a Cartesian linear motor which has been optimized for use in conjunction with a class of high-precision magnetic suspension stages for photolithography. The paper gives analytical solutions for this motor's fields, forces, commutation structure, and power dissipation; and develops results for the power-optimum thickness of the stator windings. Finally, we suggest that an equivalent Halbach geometry electromagnet may find utility in areas such as magnetically levitated trains.

2 Overview of Halbach arrays

For the present purposes, the Halbach array has the following advantages: The fundamental field is stronger by 1.4 than with a conventional array, and thus the power efficiency of the motor is doubled. The array does not require an iron backing sheet, and so can be bonded directly to non-ferrous substrates such as aluminum or ceramic which may have desirable structural properties. The magnetic field is more purely sinusoidal than that of a conventional magnet array, resulting in a simple control structure. Finally, the array has very low back-side fields, and thus requires less shielding in low-field applications.

The key concept of the Halbach array [1,2] is that the magnetization vector should rotate as a function of distance along the array so as to maximally aid the desired field distribution. In fact if the vector rotates continuously, the field on one side of the array will be identically zero, while the field strength on the other side of the array is doubled relative to an array with a sinusoidally varying purely vertical magnetization. The choice of the direction of rotation determines which side of the array is the strong side and which is the weak side.

While the ideal case calls for continuous rotation, in practice it is not possible to construct magnetic material with a continuously rotating easy axis, nor to magnetize this material in the proper pattern if it could be constructed. Thus practical implementations of Halbach arrays are assembled from blocks of magnetic material which are each uniformly magnetized in a desired axis. By proper choice of the block magnetic axes, a close approximation of the ideal case can be achieved. As mentioned previously, the Halbach arrays have geometries useful for synchronous machines in Cartesian, polar, and cylindrical coordinates. These correspond to magnet arrays for planar linear, cylindrical rotary, and cylindrical linear motors, respectively.
The finite element solution of the field pattern for a section of cartesian Halbach array is shown in Fig. 1. The arrows in each block indicate the direction of magnetization. This particular array uses four blocks per period, with the magnetization axis rotating by 90 degrees in each subsequent block. Further, the array thickness is equal to one quarter of the spatial period and thus this array is easily fabricated, since the blocks are identical and are simply rotated as the array is assembled. We later show that the fundamental field on the strong side of this array is within 90% of the field of the ideal array in which the rotation is continuous. Thus this is the magnet array we have adopted for the linear motor which drives our magnetic bearing stage.

Finite element solutions of the field patterns for two polar Halbach quadrupole arrays are shown in Figs. 2 and 3. The arrows in each block indicate the direction of magnetization. These particular arrays use eight blocks per period, with the magnetization axis rotating by 45 degrees in each subsequent block relative to the radius vector through the block center. Note that the only difference between the two arrays is in the direction of rotation of the magnetization vector. In Fig. 2 the vector rotates clockwise as the array is traversed in the clockwise sense, and the field is concentrated on the inside of the array; in Fig. 3 the vector rotates in the opposite direction and the field is concentrated on the outside of the array. Dipoles and higher-order multipoles can also be constructed in this geometry by choice of the number of rotations of the magnetization vector encountered in one circuit around the annulus. In particle accelerators, such arrays are used for focusing and otherwise manipulating the particle beam. To a motor designer it is clear that this polar array can be used as either the rotor or stator of a rotary synchronous machine.

The geometry of a magnet array suited to a cylindrical linear motor is shown in cross-section in Fig. 4. As shown in the figure, the magnetization rotates about a vector tangent to the cylinder, and is constant in $r$-$\theta$ direction and magnitude as a function of the azimuthal coordinate. The fabrication of this motor requires two types of permanent magnets which are in the shape of identical annuli, i.e., one with axial magnetization and one with radial magnetization. Both can be readily fabricated. In this case, the controlled motion is along the axis of the tube, and the stator takes the form of annular coils arranged axially along the surface of a cylinder. Such a motor can be made to be highly efficient since both cross-axis fringing fields and winding end-turns are eliminated, due to the fact that there is no transverse boundary. Again, by choice of the direction of magnetization rotation, the working field of this cylindrical array can be located on either the outside or inside of the tube, as dictated by a particular design application. When viewed in a transverse section, the field pattern for this motor is similar to that of the cartesian array and thus is not shown herein.
3 Magnetic bearing stage for photolithography

The authors have previously reported on the design of a magnetic bearing stage for precision motion control in photolithography [3–5]. The function of a photolithography stage is to position a semiconductor wafer in six degrees of freedom so as to align a die site on the wafer with the image plane of the lithographic lens in order to expose subsequent circuit layers in precise alignment. The stage must provide 200 mm travel in the plane of the wafer (X and Y axes) so that any portion of the wafer can be positioned under the lens, and must also provide approximately 300 μm travel normal to the wafer for fine focusing. In current lithography systems each die site is locally levelled and rotationally aligned to the image plane, so the stage must also provide milliradian-scale rotations. The stage must be able to rapidly move from die-site to die-site (typically 20 mm apart) and thus fast settling is important. Our design goal is to step 20 mm and settle to the required precision in under 200 msec. At a particular die site, the stage position must be maintained with a standard deviation better than 10 nm during the approximately 300 msec die-site exposure time. Thus both high speed and high precision are required.

A magnetic bearing stage is advantageous in this application, since the small focus and rotational alignment motions can be supplied for “free” given that the stage is in suspension. The magnitude of the required motions is also compatible with the constraint that the magnetic bearing air gap not vary too much (i.e., motions at the air gap should be less than about 1 mm) in order to obtain high power efficiency from the actuators.

It is possible to envision stages which have a “flying-puck” design in which a single moving part is suspended and controlled over the complete range of motion [3]. However, the actuator size and packaging constraints of such a design result in a stage which is excessively large and is inefficient in power and in the use of magnetic materials.

A more efficient design which we have adopted utilizes crossed axes, in which the base axis is a conventional mechanical linear slide (Y axis) which carries a magnetic bearing linear slide (X axis). This design yields a much more efficient motor structure and packaging factor.

Such a crossed-axis design takes advantage of the typical operation of a lithography machine. In normal operation, the stage is rastered in X across 10–20 die sites before a single step is made in Y in order to move to the next row of die sites. Thus the throughput performance of the machine is dominated by the X-positioning stage and depends much less upon the Y-positioning stage. Since the crossed-axis design allows a simple magnetic bearing structure and yields better performance and power efficiency, this is the design which we have adopted.

Given a crossed-axis design, the remainder of this paper focuses on the requirements of the X-positioning stage. These are travel of 200 mm in the X-axis, travel of 300 μm in the Y and Z axes, and milliradian-scale rotation about all three axes. These requirements have been addressed through the design of a linear magnetic bearing stage which is constrained in five degrees of freedom by variable reluctance magnetic bearings, and driven in the sixth degree of freedom by a permanent magnet linear motor. The magnetic bearing controls translations in Y and Z and rotations about all three axes. This bearing is described in [3–5] and will not be discussed further here. The use of a Halbach array in the design of the linear motor provides the focus for this paper.

To provide an indication of the positioning performance achievable with such a stage, Fig. 5 shows the X-axis step response of our first-generation magnetic linear bearing [5]. In this figure, the bearing is commanded to take a 200 nm step in the X-axis. The feedback in this axis is via a plane mirror laser interferometer, and the linear motor is digitally-controlled by a ‘386-based PC housing a laser interferometer interface card and analog output card. The 5 nm quantization of the interferometer is visible in the figure.

The bearing control is presently implemented in analog electronics; subsequent versions now in construction will use digital control in the bearing axes as well. The

\footnote{While for convenience some dimensions are given in other than meters, all variables in analytical expressions are in SI units.}
Figure 5: X-axis step response (200 nm) of first-generation linear magnetic bearing stage. Vertical axis is 50 nm per division; horizontal axis is 100 msec per division.

The step response shows that we can achieve the required settling time and position precision. However, due to bearing nonlinearities, we are as yet unable to demonstrate the required acceleration necessary to achieve a 20 mm/200 msec step-and-settle time. Additionally, the motor used in this experiment uses a conventional magnet array with pure normal magnetization, and thus is not as power efficient as a similar motor using a Halbach magnet array. These limitations are to be corrected in a second-generation stage which is now in fabrication. The motor for this stage is described in more detail below.

4 Design issues

While a Halbach array is applicable for many permanent magnet machine designs, it is particularly suited to the linear magnetic bearing stage which we have designed. To demonstrate this suitability, it is necessary to understand the issues which must be addressed by the linear motor design and to understand those features which are unique to the magnetic bearing application. These are as follows:

1. No lateral loading of linear magnetic bearing; zero force at zero current; no cogging.
2. Low total power dissipation with no power dissipation on platen; low resistance thermal path from stator to air.
3. Stator wires in fixed frame, thus no need for leads onto moving platen.
4. Independently controllable levitation and drive forces with equal control authority in both degrees of freedom; wide-bandwidth, linear control.
5. Motor electromechanics analytically tractable, so analytical optimization of geometry and analytical derivation of control laws are possible.
7. Motor able to accommodate 300 μm changes in air gap with little loss in force capability; axial travel of 200 to 300 mm.

With respect to item 1, it is highly desirable that the linear motor exert no extraneous forces on the linear bearing. That is, when the motor control currents are zero, the linear motor forces in all axes should be zero. This is in contrast with a conventional linear motor which, if it uses iron, may exert significant off-axis forces due to the attraction of the permanent magnets to the iron in the stator. The requirement in our case for low extraneous forces is due to the limited force capabilities of the magnetic bearing and to the requirement that the bearing dissipate a minimum of power. Any side loads on the bearing result in unacceptable power dissipation or overloading of the magnetic bearing. Thus any motor structure must be either balanced (two-sided), or if single-sided, then the stator must be iron-free so that it is not strongly attracted to the permanent magnet array. The requirement for no cogging (item 1) greatly eases the task of precision control, and again suggest an ironless motor design. It is easier to package a planar single-sided motor as opposed to a more conventional slotted linear motor (item 6), since a slotted motor requires greater stage height and weakens the stage with respect to resonant modes. Additionally, a planar single-sided motor with the stator windings on the fixed base presents a large-area thermal path out of the machine (item 2). Higuchi [6] has constructed a two-sided variable reluctance linear motor for a magnetic suspension semiconductor wafer transporter. However this design is difficult to package in a lithography machine. A single-sided variable reluctance motor such as used in a Sawyer motor [7] has unbalance forces approximately ten times as large as the lateral force capability of the motor and thus requires an air bearing to counterbalance these forces. However, the use of a planar air bearing eliminates the ability to move normal to the motor air gap and thus eliminates the capability of the stage to provide focus motions. Thus the variable reluctance linear motor structure has been eliminated from consideration. An induction machine of necessity dissipates power on the moving stator and thus is not considered further. On the basis of the design considerations stated above we have chosen to develop the permanent magnet linear motor which is discussed in detail in the following section.
5 Linear motor design and analysis

A simplified cross section of the second generation magnetic bearing stage is given in Fig. 6 in order to show the motor geometry. The linear motor consists of a 250 mm long cartesian Halbach array on the bottom surface of the moving platen, and a surface-wound stator of 500 mm length on the top surface of the stator base. This base is to be carried by the Y-stage, although the details are not discussed here. The motor has a depth of 150 mm into the paper.

In order to analyze the motor, we first study several possible magnet arrays and solve for their associated fields. The result is that the Halbach array field is a factor of $\sqrt{2}$ stronger than that of a conventional magnet array. We next solve for the fields due to the stator windings and then apply superposition to give the fields in the presence of both the stator and the Halbach array. Once the fields are known, the Maxwell stress tensor is used to solve for the normal and lateral motor forces, under the assumption of a two-phase sinusoidally-distributed stator. Inverting this analytical solution for the motor force yields the motor commutation laws which are used in the digital controller. Following this, the motor power dissipation is analytically determined and a power-optimal stator thickness is derived. Finally, the expected performance characteristics of our linear motor are presented.

5.1 Analytical preliminaries

To calculate the fields and thus the electromagnetic forces, the motor is idealized as a two-dimensional structure as shown in Fig. 7. Here the motor is assumed to have a depth of $w$ into the paper and to extend indefinitely in the $\pm z$ direction. Edge effects in the $y$-direction are ignored in this analysis so that a two-dimensional model is applicable. The stator is fixed in the laboratory frame $x$, $y$, $z$ (Y-slide motions are ignored herein). The primed coordinate frame $x'$, $y'$, $z'$ is fixed in the layer of magnetization, and is displaced from the unprimed frame by the vector $(x_0 + \Gamma)i_x + z_0i_z$. Throughout the following analysis vector quantities are represented by an overbar.

The magnetization layer is of thickness $\Delta$, and within this layer the magnetization is represented by the Fourier series

$$\bar{M} = \sum_{n=-\infty}^{\infty} [\bar{M}_{vn}i_x + \bar{M}_{hn}i_z] e^{-jk_nz}$$

Here $\bar{M}_{vn}$ and $\bar{M}_{hn}$ are the complex amplitudes of the $n^{th}$ vertical and horizontal Fourier magnetization components, respectively. Assuming that the spatial period of the array is $l$, then $k_n = 2\pi n/l$ is the wavenumber of the $n^{th}$ Fourier component.

The stator layer is of thickness $\Gamma$, and within this layer the current density is represented by the Fourier series

$$\bar{J} = \sum_{n=-\infty}^{\infty} \bar{J}_n y e^{-jk_nz}$$

Here $\bar{J}_n$ is the complex amplitude of the $n^{th}$ current density component. Note that the stator current is purely $y$-directed and is assumed constant in $x$ within the layer, although the analysis could be extended to address current densities which vary with $x$.

Letters (a), (b),....,(h) indicate the top and bottom surfaces of the four boundaries. Throughout this analysis, quantities with a tilde are the complex amplitudes of Fourier components; these represent temporal variations in both amplitude and spatial ($z$) phase. Variables with a superscript a,b,...n represent a quantity evaluated on the corresponding boundary. The notation and analytical approach used herein follow [8], chapters 2, 3, and 4.

5.2 Cartesian magnet arrays

In this section we compare the fields of four possible magnet array topologies shown in Fig. 8. The top row of the figure shows topologies for the ideal Halbach array (A)
and the vertical sinusoidal array (B) where the magnetization varies sinusoidally with the lateral dimension. It is not practical to fabricate either of the arrays in the top row. The bottom row shows a Cartesian Halbach array with four blocks per period and thus ninety degree rotations per block (C) and a conventional magnet array with alternating vertical magnetized blocks (D). The figure also shows the magnetic charge \( \rho_m = -\nabla \cdot \mu_0 M \) associated with each layer. Note that the Halbach arrays have charge within the layer associated with the spatially varying horizontal component of magnetization as well as the magnetic surface charge associated with the termination of vertical magnetization in all four arrays.

In order to compare the arrays, we assume that the magnetic material has a peak magnetization of \( M_0 \). Further, since the stator contains no iron, once the magnetic scalar potential \( \psi \) due to the magnet array is known on the upper (a) or lower (d) boundary it is known throughout the corresponding half-space, and thus the associated fields are also known. Thus it is reasonable to compare the array field strength through the value of the scalar potential on boundaries (a) and (d). Using the analysis presented in Appendix A, we can show that the fundamental components of the potentials of the four arrays are as follows:

\[
\begin{align*}
\psi_{IA}^d &= F(M_0, \Delta, k_1, z') \quad \psi_{IA}^c = 0 \\
\psi_{IB}^d &= \frac{1}{2} F(M_0, \Delta, k_1, z') \quad \psi_{IB}^c = -\psi_{IB}^d \\
\psi_{IC}^d &= \frac{2\sqrt{2}}{\pi} F(M_0, \Delta, k_1, z') \quad \psi_{IC}^c = 0 \\
\psi_{ID}^d &= \frac{2}{\pi} F(M_0, \Delta, k_1, z') \quad \psi_{ID}^c = -\psi_{ID}^d
\end{align*}
\]

(3)

where \( F(M_0, \Delta, k_1, z') \equiv -(M_0/k_1)(1 - e^{-k_1 \Delta}) \cos k_1 z' \) is introduced to emphasize that this function is common to all four arrays.

The first thing to notice is that the Halbach arrays have zero fundamental on the upper surface, whereas the vertical arrays have vertically antisymmetric fields. The highest potential magnitude is for the ideal Halbach array (A), whereas the vertical sinusoidal array (B) is weaker than (A) by a factor of 2. However, the block Halbach array (C) has a potential which is within \( 2\sqrt{2}/\pi \) or about 90% of the ideal case (A). Further, the block Halbach array (C) is stronger than the block vertical array (D) by a factor of \( \sqrt{2} \). For the ideal Halbach array (A) it can be further shown that there are no higher Fourier components on either surface. In the case of the block Halbach array (C), the next component on the strong side (d) above the fundamental is \( n = 5 \), whereas for the block vertical array (D), an \( n = 3 \) component is present. Thus the Halbach array yields a more purely sinusoidal field, which simplifies the task of commutation. As a final point, the term \( (1 - e^{-k_1 \Delta}) \) appearing in \( F(M_0, \Delta, k_1, z') \) is approximately equal to 0.8 for \( \Delta = l/4 \). Thus the block array which uses identical square magnets has a field strength within 80% of that obtainable from an infinite thickness array. Since the block Halbach array provides the highest fields for the arrays which can be fabricated, it is solely considered in the subsequent motor analysis.

5.3 Stator fields

Since we intend to use the Maxwell stress tensor to solve for the motor forces, it is only necessary to know the system fields along one air-gap boundary. For the present purposes boundary (d) is chosen. There are several approaches to solving for these stator fields; since this is a conventional problem, only the end results are given. Specifically, the complex amplitude of the \( n^{th} \) component of the vertical field on boundary (d) due to the stator is

\[
S_{2n}^d = \frac{j J_n}{2k_n} e^{-\gamma_n x_0 (1 - e^{-\gamma_n l})} e^{-j k_n x_0}
\]

(4)

Similarly the horizontal component is

\[
S_{2n}^h = \frac{-j J_n}{2\gamma_n} e^{-\gamma_n x_0 (1 - e^{-\gamma_n l})} e^{-j k_n x_0}
\]

(5)

where \( \gamma_n \equiv |k_n| \). Making use of \( z' = z - x_0 \), these complex amplitudes are expressed with respect to the primed frame. That is, we choose to consider boundary (d) as fixed in the magnet and thus in the primed frame.

5.4 Total fields

Given the potentials driven by the magnet array which were determined in section 5.2, the magnetic field is solved for as \( \vec{H} = -\nabla \psi \). Evaluating these fields on boundary (d) gives the complex amplitudes on the boundary which are due to the magnet, i.e., \( \vec{H}^d_{mn} \) and \( \vec{H}^d_{zn} \). The total complex amplitudes at the boundary are then given by \( \vec{H}^d_{xn} = S_{2n}^d \vec{H}^d_{xn} + M_{2n}^d \vec{H}^d_{zn} \), or upon expanding

\[
\vec{H}^d_{xn} = \frac{j J_n}{2k_n} e^{-\gamma_n x_0 (1 - e^{-\gamma_n l})} e^{-j k_n x_0}
\]

\[
+ (M_{2n} - j \gamma_n \vec{M}_{hn})(1 - e^{-\gamma_n \Delta})/2
\]

(6)
and by \( \vec{H} = S \vec{H} + M \vec{H} \), or upon expanding

\[
\vec{H} = \frac{-\vec{j}_n}{2\gamma_n} e^{-\gamma_n \varepsilon (1 - e^{-\gamma_n \varepsilon})} e^{-jk_z z_0} + (j \frac{\vec{k}_n}{\gamma_n} M_{\text{mn}} - M_{\text{hn}})(1 - e^{-\gamma_n \Delta})/2.
\] (7)

5.5 Motor forces

As presented in [8], the stress tensor for magnetically-linear materials associated with the Korteweg-Helmholtz force density is

\[
T_{ij} = \mu H_i H_j - \frac{\mu}{2} H_i H_i H_k
\] (8)

using the Einstein summation convention where since the \( k \)'s appear twice in the same term they are to be summed from one to three. The force acting on a volume of the magnet array is given by the integral of the stress tensor over the surface of the volume. For a spatially-periodic structure, the integration is simplified if the volume encloses an integer number of periods. In this case the components on the \( z \)-faces of the volume cancel due to symmetry. We can consider the upper surface of the volume to extend to infinity, where the fields are zero and thus the only contribution is along the bottom surface which we take to lie on boundary (d).

If the lower surface encloses an integer number of periods and is of area \( A \), then the \( z \)-directed force acting on the enclosed section of magnet array is given by

\[
F_z = -A \langle \sigma \vec{T} \rangle_z = -A \mu_0 \langle H_x^2 H_z - H_z^2 H_x^2 \rangle_z,
\] (9)

and the \( x \)-directed force acting on the enclosed section of magnet array is given by

\[
F_x = -A \langle \sigma \vec{T}_{xx} \rangle_z = -A \mu_0 \langle H_x^2 H_x^2 \rangle_z,
\] (10)

where the angle bracket expression \( \langle \cdot \rangle_z \) indicates the spatial average of the quantity enclosed by the brackets. The minus sign appears because the bottom surface has an outwardly-directed normal in the \(-x\)-direction. A useful identity is the spatial averaging theorem ([8], section 2.15)

\[
\left\langle \sum_{n=-\infty}^{\infty} A_n e^{-jknz} \sum_{m=-\infty}^{\infty} B_m e^{-jk_mz} \right\rangle_z = \sum_{n=-\infty}^{\infty} A_n \hat{B}_{-n} = \sum_{n=-\infty}^{\infty} \hat{A}_n \hat{B}_n^*.
\] (11)

Although the analysis can be carried out more generally, in the following we assume that the stator currents are sinusoidally distributed with a fundamental period of \( \varepsilon \) and thus that \( \vec{j}_n \) is equal to zero for \( n \neq \pm 1 \). Specifically, we let \( \vec{j}_1 = \vec{j}_a + j \vec{j}_b \), and \( \vec{j}_{-1} = \vec{j}_a - j \vec{j}_b \). That is, \( 2j_a \) is the peak phase A current density and \( 2j_b \) is the peak phase B current density. The above assumption is reasonable since it is primarily the fundamental field components which are responsible for force production. Further, in application, the spatial currents are driven to resemble a sinusoid and thus the assumption is accurate for our purposes. Under this assumption, applying (11) to (9) and (10), and using (6) and (7) yields, after some algebra, the forces acting on one spatial period of the magnet array as

\[
\begin{bmatrix}
F_{x,\lambda} \\
F_{y,\lambda}
\end{bmatrix} = \mu_a M_o G e^{-\gamma_z z_0} \begin{bmatrix}
-\sin \gamma_z z_0 & \cos \gamma_z z_0 \\
\cos \gamma_z z_0 & \sin \gamma_z z_0
\end{bmatrix} \begin{bmatrix}
J_a \\
J_b
\end{bmatrix}
\] (12)

where \( F_{x,\lambda} \) and \( F_{y,\lambda} \) are the \( x \)-directed and \( z \)-directed forces per spatial wavelength, respectively. Here \( \mu_a M_o \) is the remanence of the permanent magnets (\( \approx 1.2 \) T for neodymium-iron-boron). The constant

\[
G = \sqrt{2w l^2} \pi \gamma_z (1 - e^{-\gamma_z \varepsilon} (1 - e^{-\gamma_z \Delta})
\] (13)

contains the effects of the motor geometry. The \( \varepsilon \) and \( z \) dependencies have been explicitly retained since these variables represent motion of the magnet array relative to the stator.

In an implementation, densities \( J_a \) and \( J_b \) are established by two sinusoidally-distributed windings which we can model as having a peak turns density of \( \gamma_0 \) turns per meter squared and terminal currents \( I_a \) and \( I_b \), respectively. In this case, \( J_a = I_a / \gamma_0 \) and \( J_b = I_b / \gamma_0 \). However, for present purposes, the analysis will be carried out in terms of current densities.

For the purposes of control, the motor commutation laws are derived by inverting (12) to yield

\[
\begin{bmatrix}
J_{xa} \\
J_{yb}
\end{bmatrix} = \frac{e^{\gamma_z z_0}}{\mu_0 M_o G N_m} \begin{bmatrix}
-\sin \gamma_z z_0 & \cos \gamma_z z_0 \\
\cos \gamma_z z_0 & \sin \gamma_z z_0
\end{bmatrix} \begin{bmatrix}
F_{x,\lambda} \\
F_{y,\lambda}
\end{bmatrix}
\] (14)

where \( N_m \) is the number of spatial periods \( l \) of the magnet array which interact with the stator. Here \( F_{x,\lambda} \) and \( F_{y,\lambda} \) are signals that exist within the controller and represent the forces the controller is requesting to act on the entire motor in the \( x \) and \( z \) directions respectively. The signals \( J_{xa} \) and \( J_{yb} \) are the current densities which are calculated to achieve the desired forces. These are scaled to take account of the winding density \( \gamma_0 \) before being output to set the phase currents \( I_a \) and \( I_b \), which are assumed to be controlled by current drive amplifiers. If the analytical model (12) is accurate, then the commutation laws (14) linearize and decouple the plant. The
controller which sets \( F_{sd} \) and \( F_{ed} \) is then a simple decoupled lead/lag compensator. In practice, since the bearing controls all lateral displacements, we have been setting \( F_{sd} = 0 \) (open-loop control) and only actively controlling lateral motion through \( F_{ed} \). Thus in our present experiments the controller is single-input–single-output. The performance which can be achieved by such a control strategy was shown earlier in Fig. 5.

5.6 Power dissipation

In precision machines, power dissipation is a key factor due to the fact that materials change dimensions significantly even with small temperature variations. Thus a focus of our design efforts has been to build a highly power-efficient machine. One advantage of an analytical solution for the motor force characteristics is that this solution allows us to also analytically determine the force-power characteristics. This solution makes clear how the motor should be designed and scaled to maximize power efficiency. With this solution in hand, we can further derive a power-optimal thickness \( \Gamma \) for the stator.

The power dissipation density in a region of conductivity \( \sigma \) carrying a current density \( J \) is \( J^2/\sigma \). Thus the power density in the stator is \( P_s = [(J_a - j J_b) e^{j\pi z} + (J_a + j J_b) e^{-j\pi z}]^2/\sigma \). This expression is averaged on \( z \) and then multiplied by the volume of one spatial period to yield \( P_s = \frac{2w l \Gamma (J_a^2 + J_b^2)}{\sigma} \), where \( P_s \) is defined as the power dissipation per wavelength in the working region of the motor. By (14), the current density term \( (J_a^2 + J_b^2) \) can be expressed in terms of the commanded forces. This allows us to solve for the total motor power dissipation \( P_t \) as

\[
P_t = \frac{3N_s \pi^4 \Gamma B^2 \gamma x_0 (F_{sd}^2 + F_{ed}^2)}{N_m \sigma (\mu_0 M_0)^2 2w l^3 (1 - e^{-\gamma l})^2 (1 - e^{-\gamma l})^2} \tag{15}
\]

where \( N_s \) is the total number of spatial periods \( l \) in the stator. The factor of three appears because we assume that the motor dissipates three times more power than that given by the ideal calculation, due to non-ideal packing factor in the windings, significant winding length in the end turns, and to fringing fields.

Note that this expression is independent of motion in \( z \) and depends upon the sum of the squares of the commanded forces. Note also that the power dissipation drops as the square of the permanent magnet remanence. Thus we design with high-remanence magnetic material. We can also see clearly here the confirmation of the result stated earlier, i.e., that since the Halbach array yields a field stronger by \( \sqrt{2} \) relative to a conventional magnet array, the power efficiency of the motor is doubled.

Power grows exponentially with \( x_0 \), and thus it is important to run the motor at a small air gap. However, the problem is not severe, since the characteristic dimension to which \( x_0 \) is compared in the exponent is \( l \). For instance if \( l = 50 \text{ mm} \), then at an air gap of \( x_0 = 0.5 \text{ mm} \), the power dissipation is only 14\% higher than at zero air gap. Thus the motor force is not sensitive to vertical focusing motions, and the design satisfies our earlier-stated criteria (item 7).

We solve for the power optimal stator thickness as follows. The term in (15) that depends on \( \Gamma \) is \( \Gamma / (1 - e^{-\gamma l})^2 \). Setting the partial of this term with respect to \( \Gamma \) equal to zero yields a transcendental equation \( 1 + 2\gamma l = e^{\gamma l} \). Solving this numerically yields the minimum power solution \( \gamma_1 l \approx 1.25 \). Now, since \( \gamma_1 = 2\pi / l \), we find \( \Gamma \approx l / 5 \) as the power optimal stator thickness.

As another way to minimize power, consider the motor scaling laws in power and force. Specifically, since power increases as the square of current and force increases linearly with current, if the motor area is increased by a factor of \( n \), then the power required to achieve a given force is reduced by this factor. This result is apparent in (15), since if \( N_m \) and \( N_s \) are scaled by \( n \), then the motor power dissipation \( P_t \) drops by this same factor. Following this reasoning, we have designed a large motor which takes advantage of the significant planar area available on the bottom surface of the lithography stage. This motor is described in more detail below.

5.7 Example motor design

As an example of the achievable performance, consider a motor with the following specifications: magnet area: 0.15 m wide by 0.25 m long, stator area: 0.15 m wide by 0.5 m long, \( w = 0.15 \text{ m} \), \( l = 0.05 \text{ m} \) (five spatial periods in magnet array, \( N_m = 5 \), ten spatial periods in stator, \( N_s = 10 \), \( \Gamma = l / 5 \) (power optimum), \( \Delta = l / 4 \) (square magnet blocks), \( x_0 = 2.5 \times 10^{-4} \text{ m} \), \( \sigma = 5.6 \times 10^{7} \text{ S/m} \) (copper), and \( \mu_0 M_0 = 1.2 \text{ T} \) (NdFeB magnets). Substituting these values into (15) yields a motor power coefficient of about 2.6 \times 10^{-3} \text{ W/N}^2. For example, at 50 N force, the motor dissipates a total of 6.5 W.

To put this result in context, consider a 200 msec step-and-settle time with a 15 kg platen. Under this assumption, we calculate a power dissipation of 2.3 W while the stage is moving. Since the stage is held fixed for more than half the time while an exposure is in progress, the average power will be lower than 1.1 W. We estimate that given the low-resistance thermal path out of the stator, the winding temperature will rise less than 0.1° C. This shows that the motor meets the criteria for low power (item 2).

6 Maglev train electromagnets

Since permanent magnets can be thought of in terms of equivalent current sheets, there exists an electromagnetic
analog of the Halbach array, in which circulating currents with proper orientation replace the rotated permanent magnet blocks. Such an electromagnetic array has the advantage that its field can be constrained primarily to one side of the array. This topology has potential use in systems where low fields on the back side of the electromagnet are important, for instance in the superconducting electromagnets of maglev trains. Since there is no fundamental field on the back side of the array, and since the higher harmonics decay rapidly with distance from the array, it should prove far easier to shield the passenger compartment of a maglev train which uses a Halbach-type electromagnet vis-a-vis one which uses a conventional magnet [9]. This issue bears further study and can be addressed using analyses similar to those given herein.

7 Conclusions
This paper has presented the design of synchronous permanent magnet motors utilizing the Halbach array topology. The design of a cartesian linear motor has been developed in depth. This motor is optimized for use with a novel magnetic suspension linear bearing which is designed for photolithography. We have demonstrated that the motor is highly power efficient and have presented the key motor design constraints and performance characteristics. A second-generation magnetic suspension stage using this motor design is currently in construction. The insight obtained from studying the Halbach permanent magnet array geometry may prove useful in other applications such as maglev trains.

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Bibliography

Appendix A: Field Solution Details
This section provides the details which support the magnet array comparison presented in section 5.2. Given the magnetization (1), we seek a solution for the scalar potential \( \psi \) on the boundaries (a)–(d). Since superposition applies, the stator windings are ignored throughout the remainder of this appendix.

In the following, we assume that the potentials and fields are given by a Fourier series, i.e.,

\[
\psi = \sum_{n=-\infty}^{\infty} \tilde{\psi}_n e^{-jn\pi x} \quad (16)
\]

and

\[
\bar{H} = \sum_{n=-\infty}^{\infty} \left[ \bar{H}_{zn} n x + \bar{H}_{zn} n x \right] e^{-jn\pi x}. \quad (17)
\]

The vertical component of magnetization terminates abruptly on the upper and lower boundaries and is constant in the volume; this creates surface charges on the upper and lower boundaries

\[
\sigma^b_s = -n \cdot [\mu_0 \bar{M} - \mu_0 \bar{M}] = \mu_0 \sum_{n=-\infty}^{\infty} \bar{M}_n e^{-jn\pi x}. \quad (18)
\]

and \( \sigma^s_c = -\sigma^b_s \), respectively, where \( n \) is the surface unit normal. These surface charges are addressed through the jump conditions

\[
\bar{H}_{zn} = \bar{H}_{zn} = \bar{M}_n
\]

and thus do not affect the solution within the magnetization layer.

Since the magnetization has a spatially-varying horizontal component, the volume contains a polarization
charge given by

\[ \rho_m = -\nabla \cdot \mu_0 M = \mu_0 \sum_{n=-\infty}^{\infty} j k_n \tilde{M}_{hn} e^{-j k_n z'}. \quad (20) \]

Within the volume we seek a solution satisfying \( \nabla \times \mathbf{H} = 0 \), and thus \( \mathbf{H} = -\nabla \psi \). Thus it also holds that

\[ \tilde{H}_{zn} = j k_n \tilde{\psi}_n. \quad (21) \]

The field must additionally satisfy

\[ \nabla \cdot \mu_0 \mathbf{H} = -\nabla \cdot \mu_0 \mathbf{M}. \quad (22) \]

Since the volume contains charge, we take the approach of determining a particular solution \( \tilde{H}_p = -\nabla \psi_p \) and a homogeneous solution \( \tilde{H}_h = -\nabla \psi_h \) where \( \nabla \cdot \mu_0 \mathbf{H}_p = -\nabla \cdot \mu_0 \mathbf{M} \) and \( \nabla \cdot \mu_0 \mathbf{H}_h = 0 \).

In light of (20)–(22) the volume charge imposes the constraint on the particular solution

\[ \tilde{\psi}_n = \frac{j \tilde{M}_{hn}}{k_n}. \quad (23) \]

If we further consider that the total boundary potentials \( \tilde{\psi}^b_n \) and \( \tilde{\psi}^c_n \) are given, then the homogeneous solution is used to match these potentials, i.e.,

\[ \tilde{\psi}_n = (\tilde{\psi}^b_n - \frac{j \tilde{M}_{hn}}{k_n} \sinh k_n x') \frac{\sinh k_n \Delta}{\sinh k_n \Delta} \\
- (\tilde{\psi}^c_n - \frac{j \tilde{M}_{hn}}{k_n} \sinh k_n (x' - \Delta)) \frac{\sinh k_n \Delta}{\sinh k_n \Delta}. \quad (24) \]

Within the magnetization layer \( \tilde{\psi}_n = \tilde{\psi}_n + \tilde{\psi}_h \). Then, since \( \tilde{H}_z = -\partial \tilde{\psi} / \partial x' \), we find that

\[ \tilde{H}_{zn} = \begin{bmatrix} \tilde{H}^b_{zn} \\ \tilde{H}^c_{zn} \end{bmatrix} = \begin{bmatrix} k_n \begin{bmatrix} - \coth k_n \Delta & \frac{1}{\sinh k_n \Delta} \\ \sinh k_n \Delta & - \coth k_n \Delta \end{bmatrix} & \begin{bmatrix} \tilde{\psi}^b_n \\ \tilde{\psi}^c_n \end{bmatrix} \\ + j \begin{bmatrix} \frac{\cosh k_n \Delta - 1}{\sinh k_n \Delta} & \frac{\cosh k_n \Delta - 1}{\sinh k_n \Delta} \end{bmatrix} \tilde{M}_{hn} \end{bmatrix}. \quad (25) \]

Evaluating (25) at \( x' = 0 \) and \( x' = \Delta \) yields

\[ \begin{bmatrix} \tilde{H}^b_{zn} \\ \tilde{H}^c_{zn} \end{bmatrix} = k_n \begin{bmatrix} \begin{bmatrix} \coth k_n \Delta & -\frac{1}{\sinh k_n \Delta} \\ \sinh k_n \Delta & \coth k_n \Delta \end{bmatrix} & \begin{bmatrix} \tilde{\psi}^b_n \\ \tilde{\psi}^c_n \end{bmatrix} \\ + j \begin{bmatrix} \frac{\cosh k_n \Delta - 1}{\sinh k_n \Delta} & \frac{\cosh k_n \Delta - 1}{\sinh k_n \Delta} \end{bmatrix} \tilde{M}_{hn} \end{bmatrix}. \quad (26) \]

This result gives the transfer relation between the complex amplitudes of the boundary potentials \( \tilde{\psi}^b_n \) and \( \tilde{\psi}^c_n \) and the normal magnetic field \( \tilde{H}^b_{zn} \) and \( \tilde{H}^c_{zn} \) given the volume source \( \tilde{M}_{hn} \). The first term on the right represents the homogeneous solution and the second term on the right represents the particular solution. This transfer relation may be used to represent any region with a Fourier-transformable horizontal magnetization which is not a function of the vertical coordinate.

Since the bounded spaces at (a) and (d) are half-infinite, the transfer relations of [8], pg. 2.33, simplify to

\[ \tilde{H}^a_{zn} = \gamma_n \tilde{\psi}^a_n \\
\tilde{H}^d_{zn} = -\gamma_n \tilde{\psi}^d_n. \quad (27) \]

Further, since the boundaries are current-free, the potential is continuous, i.e.,

\[ \tilde{\psi}^a_n = \tilde{\psi}^b_n \]
\[ \tilde{\psi}^c_n = \tilde{\psi}^d_n. \quad (28) \]

Equations (19), (26), (27), and (28) form a set of eight equations in eight unknowns which allow us to solve for the desired potentials on boundaries (a) and (d). After some manipulation, the result is

\[ \tilde{\psi}^a_n = \frac{\tilde{M}_{vn}}{2 \gamma_n} (1 - e^{-\gamma_n \Delta}) + \frac{j \tilde{M}_{hn}}{2 k_n} (1 - e^{-\gamma_n \Delta}) \quad (29) \]

and

\[ \tilde{\psi}^d_n = \frac{-\tilde{M}_{vn}}{2 \gamma_n} (1 - e^{-\gamma_n \Delta}) + \frac{j \tilde{M}_{hn}}{2 k_n} (1 - e^{-\gamma_n \Delta}). \quad (30) \]

All that remains is to determine the Fourier components associated with each magnet array. These are given by the analysis integrals

\[ \tilde{M}_{vn} = \frac{1}{L} \int_0^L M_o e^{j k_n z'} dz' \quad (31) \]

and

\[ \tilde{M}_{hn} = \frac{1}{L} \int_0^L M_h e^{j k_n z'} dz' \quad (32) \]

where \( M_o \) and \( M_h \) are the vertical and horizontal components of the magnetization within the array. Carrying out this analysis and then substituting into (29) and (30) yields the results given in section 5.2.