**Capacity Definitions of Composite Channels with Receiver Side Information**

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**Shannon Capacity Definition for General Channels and Limitations**

- **Shannon Capacity for General Channels:**
  \[ C = \max \{ I(X;Y) \} \]

- **Limitations of Shannon Capacity Definition:**
  - Choice of codebook
  - A single encoder/decoder under Shannon definition
  - Reasonable to assume a single encoder without CSIT
  - Choice of different decoders with CSIT?

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**Alternative Definition: Outage Capacity**

- **Outage Capacity:**
  - Decode portion (1-q) of the time
  - In non-outage states, transmit rate \( R_t = \text{receive rate} \ R_r \)

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**Expected Capacity**

- **Broadcast Channel Codes for Expected Capacity**
  - Channel states: virtual multiple receivers
  - BC strategy for slow fading channels [Shamai 2003]
  - Gilbert-Elliott channel, BSC with random crossover probability

- **Expected Capacity with Outage**
  \[ C^* = \max \{ E[I(X;Y)] \} \]
  \[ C^*_o = \max \{ E[I(X;Y)] \} \]

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**Example 1: Gilbert-Elliott Channel with Different Configurations**

- **Case 3: Stationary and Nonergodic**
  - Initial state chosen according to \( (\xi_0, \xi_1) \) in fixed states
  - Shannon Capacity \( C = C_{3,r} + h(p_r) \) binary entropy
  - Outage capacity \( C_{3,r} = C_{3,r} + h(p_r) \)
  - Expected capacity \( \max (h(p_r), \xi_0) \)
  - Loss incurred from lack of CSIT

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**Example 2: Binary Symmetric Channel with Random Crossover Probabilities**

- **Random crossover probability \( p \) with distribution \( F(p) \), \( 0 < p < 1/2 \), corresponding cdf \( \tilde{F}(p) \)
- Shannon capacity \( C = 1 - h(p) \), where \( h(p) = \inf \{ F(p) = 1 \} \)
- Outage capacity, \( C^*_o = \max \{ h(p) \} \)
- Expected capacity \( C^* = \max \{ h(p) \} \)

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**Future Work: Joint Source-Channel Coding**

- New end-to-end performance metric: expected distortion
- Multi-resolution source code (SC) combined with BC channel code (CC): rates in BC region to min expected distortion
- Distortion with multi-resolution SC: Gaussian source
  - Successively refinable, bounds exist for general sources
  - Transmit over two parallel non-ergodic links [Effros et al 04]:
    - compare with multi-description SC and time-sharing CC

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**Non-ergodic sources**

- Ergodic source/channel separation: simple interface H+C
- Source/Channel codes separable without loss for some notion of capacity and some end-to-end performance metric
- In case separation fails, choice of interface? Performance vs. complexity trade-off

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**Diagram:**

- Channel diagram
- Encoder/Decoder blocks
- Shannon capacity definitions
- Outage capacity definitions
- Expected capacity definitions
- Example channel configurations
- Random crossover probabilities
- Numerical examples