Rolling Horizon Control for Networks with Random Link Capacities

Stephen Boyd   Argyris Zymnis   Dan O’Neill   Andrea Goldsmith
Electrical Engineering Department, Stanford University

ITMANET PI meeting 01/07
The problem

- multi-period network resource allocation
- unicast, split flows
- stochastically varying link capacities
- buffer limits at nodes
- linear time-varying utilities

roughly: we’re doing (centralized) multi-period joint routing/scheduling
Variables, constraints, objective

- **specification**: graph incidence matrix, sources, destinations, buffer size limits, input flow limits, capacity evolution model
- **variables**: flows, buffer sizes, input and output flows
- **constraints**: flow conservation, nonnegativity, buffer limits, link capacity, source buffer limit, source limits
- **objective**: linear in output flows with time-varying weights
Approaches

- **prescient relaxation**
  - ignore causality
  - problem becomes large LP
  - empirical utility mean gives bound on expected utility

- **optimal control**
  - current control function of current and past states
  - easily described by the Bellman recursion
  - hard to compute in general

- **rolling horizon**
  - assume future capacity equal to conditional mean
  - solve large LP to get full plan
  - use only first action
• 10 nodes, 19 links, 2 flows (A and B)
• we consider $T = 30$ steps
• buffer limits $Q_{\max} = 1.5$; input flow limit $s_{i}^{\text{in},\max} = 2$
Markov link capacity model

- three states: good \( (c = 3) \), OK \( (c = 1) \), bad \( (c = 0.1) \)
- link capacities evolve independently
- mixing time about 3 periods
- equilibrium distribution is 0.3, 0.5, 0.2; average capacity is \( \bar{c} = 1.42 \)
- all links start in OK state
Utility weights

- Flow A somewhat time-critical; flow B is best-effort
Simulation - Utility distributions

upper bound: $\mathbb{E} U \leq 92$, prescient: $\mathbb{E} U \approx 75$, rolling horizon: $\mathbb{E} U \approx 71$
Simulation - Utility gap distribution

distribution of \((U_{\text{pre}} - U_{\text{rh}})/U_{\text{pre}}\); average is 5%
Cumulative output flow (realization 1)

Prescient

Rolling-horizon

$\text{Cumulative output flow}$

$\text{A}$

$\text{B}$

$t$

ITMANET PI meeting 01/07
Central link flow and buffer size (realization 1)

ITMANET PI meeting 01/07
Extensions

‘straightforward’:

- general concave utilities
- random (e.g., Markovian) utility weights
  (e.g., flows randomly transition from ‘best-effort’ to ‘urgent’ and back)
- more general (non-Markovian) link state model
- changing source/destination nodes for flows
- joint resource allocation (bandwidth, power, . . . )
- fixed-route and multi-route flows
- multi-cast flows with fixed routes (trees)
- comparison with existing (distributed) protocols and methods

more challenging: distributed rolling-horizon methods