Semiclassical processes in a high-frequency field

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The increase in the probability for a classically forbidden process under the action of a time-varying perturbation is investigated. As is well known, semiclassical tunneling through a potential barrier $V(x)$ can be described in terms of classical trajectories satisfying Newton's equation, which, moreover, leads to the concept of motion in imaginary time. A homogeneous alternating field $\mathbf{E} \cos \omega t$ is transformed, when $t$ is replaced by $i\tau$, into a field $\mathbf{E} \cosh \omega \tau$ that increases with $\tau$, the characteristic $\tau$ values being determined by the period during which the motion takes place in the classically forbidden region. As a result, the effective field determining the change in the index of the tunneling exponential function is of the order of $\mathbf{E} \exp(i\tau \epsilon)$, where $\epsilon$ is the imaginary part of the time it takes the particle to get to a singular point of the potential after emerging from under the barrier. A phenomenological picture of tunneling in an alternating field is described, and a general approach to the computation of the probabilities for semiclassical processes occurring under non-steady-state conditions is presented. The probabilities for penetration through a potential barrier, decay of a metastable state, and above-barrier reflection are found in the weak-field limit. The complete solution to the problem of tunneling through a triangular potential barrier (field emission) in an arbitrary alternating field is presented. The simplest band-structure model for a semiconductor is used to investigate the interband breakdown in an arbitrary field $\mathbf{E} \cosh \omega \tau$ (the nonlinear Franz-Keldysh effect). The possibility of an experimental observation of the investigated phenomena is discussed.

1. INTRODUCTION

The processes of subbarrier transmission and above-barrier reflection, which are forbidden by classical mechanics, acquire finite probabilities when the quantum effects are taken into account. As a rule, these probabilities decrease exponentially as the barrier thickness to particle wavelength ratio increases. The computation of the probability for a classically forbidden process has a certain peculiarity from the mathematical standpoint: there necessarily arises here the concept of motion in imaginary time or along a complex trajectory. This characteristic of semiclassical processes makes their computation quite difficult, and because of this no detailed investigation of, in particular, the effect of a variable perturbation on classically forbidden processes has thus far been published. The problems that have been solved either assume that the variable perturbation is weak, as obtained in the case of the Franz-Keldysh effect, or are limited by the stipulation that the static potential is a short-range one, a condition which, fortunately, turns out to be adequate for the investigation of the practically important problem of many-photon ionization of an atom. Thus, a large number of phenomena that arise when semiclassical processes occurring in realistic potential fields are exposed to the nonlinear action of a high-frequency field have yet not been investigated.

In the present paper we consider the effect of an alternating field on the quantum tunneling of a particle through a potential barrier, interband tunneling in a semiconductor, and above-barrier reflection. These effects are encountered in field emission, interband breakdown, charge exchange between deep-lying impurity centers in semiconductors, tunneling chemical reactions, and the destruction of the adiabatic invariants in classical mechanics. In all situations of this kind a variable perturbation substantially increases the probabilities for the forbidden processes.

Let us illustrate the subject of the present paper by the process of tunneling. If the amplitude of the alternating field is small, then the passage through the barrier will be determined largely by ordinary tunneling, and the alternating field can be taken into account within the framework of perturbation theory. This means that the probability for tunneling accompanied by absorption of one or several photons is small compared to the probability for tunneling in zero field. But if the alternating-field strength exceeds a certain value, then the tunneling will be insignificant, and the passage will occur as a result of the absorption by the particle of that number of photons which it needs in order to get to the top of the potential barrier.

These limiting cases are separated by a broad range of alternating field-strength values at which the passage through the barrier is a process of the mixed type, in the sense that it is advantageous for the particle to absorb a certain number of field quanta so as to tunnel in a higher energy region, where the barrier is more transparent. The optimum number of absorbed quanta is determined in this case by the competition between the growth of the tunneling probability and the decrease of the absorption probability as the number of quanta increases. This type of problem is solved in Ref. 9 in the weak alternating field approximation. It is shown there that the
amplitude of the alternating field \( E \) on the motion of the particle in the forbidden region. Thus, in the high-frequency limit \( \omega > 1 \), the barrier-penetrating probability depends anomalously strongly on the amplitude of the field. The most promising is in respect of an experimental observation of the indicated effects are Josephson junctions, for which we have theoretically investigated the voltage-state lifetime in a weak alternating field. In fact, the spin that the problem under discussion is a pressing one, in view of the latest experimental advances in the study of the voltage states of Josephson junctions, the question of the effect of an alternating field on the tunneling processes has almost not been touched upon in the literature. In Refs. 17-19 the effect of an alternating field on the motion of a particle in the classically allowed region is considered. In Ref. 20 an attempt is made to take into account the effect of an alternating field on subbarrier motion, but the general arguments do not lead to specific results.

The purpose of the present paper is to investigate semiclassically processes in a high-frequency field, which is not necessarily considered to be weak. In the next section we qualitatively consider tunneling in an alternating field, and determine the typical orders of magnitude of the quantities involved in the process. In Sec. 3 we formulate the problem in a relatively general situation, and indicate a procedure for solving it with the use of the method of complex trajectories. Newton's equation in variable and spatially inhomogeneous field is considered. Finally, in Sec. 4 there is a qualitative estimate of the dominant part of the paper is devoted to the analysis of the number of specific problems. In Sec. 4 we investigate the effect of an alternating field on the tunneling process. As a result of the analysis of the semiclassical approximation requires that the indicated correction be much greater than unity, so that the transition probability should increase by several orders of magnitude. To elucidate more fully the physical picture of the phenomenon in question, we consider the effect of spatially inhomogeneous perturbations in the same linear approximation. In Sec. 5 we study the decay of metastable states in an alternating field, and show that the oscillating dependence of the argument of the tunneling exponential function on the frequency is connected not only with the normal classical, but also with a specific quantum, resonance whose frequency is determined by the particle motion in the forbidden region. In Sec. 6 we briefly investigate the effect of an alternating field on above-barrier reflection. As a result of the analysis of the exponential function for the terms nonlinear in the field is possible only for relatively small potentials; therefore, the section we shall not generalize the results obtained in Refs. 10 and 11 for a nonadiabatic potential to the nonlinear case. Below we present the solution to the nonlinear problem in two situations. In Sec. 7 we obtain and investigate in detail the exact solution to the problem of tunneling through a triangular barrier in an alternating field. These results have a direct bearing on the phenomenon of field emission. With the aid of the simplest band-structure model for a semiconductor, the contribution in Sec. 8 is detailed investigation of the interference breakdown in constant and alternating electric fields (the nonlinear Franz-Keldysh effect). In the Conclusion we discuss the results obtained in the paper and the possibility of their experimental observation.

2. QUALITATIVE ANALYSIS

Let us consider the problem of subbarrier tunneling in a uniform alternating field from a phenomenological point of view in order to elucidate the physics of the matter without laying any claims to quantitative results. We shall assume that a particle of energy \( E \) is incident from the left on a potential barrier of height \( V(x) \) of the type shown in Fig. 1. In the absence of an alternating field the probability for penetration through the barrier is, with exponential accuracy, equal to \( \exp(-A_0(E)/\hbar) \), where \( A_0 \) is the imaginary part of the corresponding action. In the general case, for a barrier of width of order \( a \) and for \( E \) below the barrier height \( A_0 \) becomes comparable to \( \hbar \), i.e., the case in which \( \hbar \) is of the order of the height of the barrier. Of course, this means that in the case of the action \( A_0 \ll \hbar \), where the amplitude of the field is small compared to the height of the potential barrier and the initial energy of the particle: \( \pi V, E \). Taking into account the relation \( A_0 \approx \pi V/2E \), where \( \hbar \) is the imaginary time of the motion under the barrier between the turning points, we obtain in place of (1) the expression

\[
D(E) = D(0) [1 + (\pi V/2E) \exp(2\pi V/2E)].
\]

3. SEMICLASSICAL DESCRIPTION OF TUNNELING IN AN ALTERNATING FIELD

Let us proceed to the derivation of the general expression for the probability for the passage of a particle through an arbitrary time-dependent semiclassical potential-barrier \( V(x, t) \). As is well known, in the semiclassical limit the wave functions can be sought with exponential accuracy in the form

\[
\psi(x, t) = \frac{1}{\sqrt{2\pi}} \exp[i(x^2/2\hbar - \int dx V(x, t))],
\]

where \( \phi(x) \) is the classical action, and \( x \) and \( \dot{x} \) lie on the particle's classical trajectory, which can be found from Newton's equation

\[
m\ddot{x} + i\hbar x = \frac{\partial}{\partial t} \frac{\delta \mathcal{S}}{\delta \dot{x}}(x, t, \dot{x}, \dot{\dot{x}}) = 0.
\]

Let the particle be incident on the barrier from the left. The problem is to find a relation between the values of the wave function at points \( x_1 \) and \( x_2 \) lying on opposite sides of the barrier. The forbidden regions of the tunneling process in the classical mechanics implies, however, that there does not exist an ordinary trajectory connecting such points. For this reason, we shall consider the trajectories in simple terms of the contour \( C \), as indicated in Fig. 2. On the symmetrical located contour \( C^* \), we have \( x(t) = x(t^*) \). The right on the contour \( C \), the quantities \( x, \dot{x}, \ddot{x} \) are real, lie to the right of the barrier, and the solution to Eq. (8) depends on two arbitrary real parameters. We shall assume that the particle emerges from under the barrier at the moment of time \( t_1 \) at the point \( x_1 \). This means that \( x(t_1) = x_1 \). The two roots \( x(t_1) \) can be taken of an imaginary time \( t_1 \) at which the particle emerges from under the barrier.

Let us assume that the nonstationary part of the potential

\[
\tilde{V}(x, t) = V(x, t) - \int_{-\infty}^{t} dt' \langle \hat{\psi}^\dagger(x, t') \hat{\psi}(x, t') \rangle
\]

is represented in the form of a product of the corresponding functions for absorption and tunneling. Nevertheless, for potential barriers having artificial singularities, for example, such that the quantities \( x(t) \) are not satisfied in the region below, exact in the quantum sense as well. Only the regions of applicability and the numerical coefficients of \( V \) in the various limiting cases are determined more accurately. Thus, the correctness of the expression (4) for the tunneling and absorption

\[
\tilde{V}(x, t) = V(x, t) - \int_{-\infty}^{t} dt' \langle \hat{\psi}^\dagger(x, t') \hat{\psi}(x, t') \rangle
\]

bars can be easily verified.
tial as at least adiabatically switched off at \( t = 0 \), \( \infty \), that \( \mathbf{V}(x) = \mathbf{V}(E) \). Then at points far to the left on the contour \( C \), the function \( \mathbf{x}(t) \) is a solution to the steady-state equation (6), and depends on two parameters, which we can take to be the time shift \( \sigma \) and the conserved quantity \( E = \mu / dx/dt + F(x) \), \( F(x) = \int_0^x \mathbf{V}(y) dy \), \( x = \sqrt{2m(E+\mathbf{V}(x))} \), respectively. In this case it is clear that the quantities \( E \) and \( t \) are, generally speaking, complex, and depend on the point \( x \) and instant \( t \) at which the particle emerges from under the barrier.

The physically justified formulation of the problem consists in prescribing the initial particle energy \( E \) and measuring the particle flux emerging from under the barrier at the instant \( t \). In the general case there are no grounds for assuming that the trajectory \( \mathbf{x}(t) \) defined by the real parameters \( E \) and \( t \), it the latter enters the problem through the condition \((dx/dt)_0 = 0\), will itself be real. A more detailed analysis of this shows, however, that the time-averaged probability for semiclassical processes is determined solely by the real trajectories. The point is that the imaginary part of the action \( A(x) = 2i\mathbb{I}N(x) \) obtained from the Hamilton-Jacobi equation is, on the basis of the semiclassical condition, large, and its variations should also be much greater than unity. Accordingly, the particle should pass through the barrier at instants lying in a narrow neighborhood of that instant at which the function \( A(t) \) has its minimum value. With allowance for the condition \((dx/dt)_0 = 0\), the condition for \( A(x) \) to be the minimum value of \( A(t) \) has the form

\[
\delta A/\delta x|_{x=0} = \partial \ell/x \partial x + \mathbf{V}(x) \cos \mathbf{V} + \partial \ell/\partial t = 0,
\]

for which the reality of \( x \) follows. Below we limit ourselves to the consideration of the minimum values of the function \( A(t) \), and we shall therefore consider the trajectory \( x(t) \) to be real. The integration contour \( C \) in this case consists of a vertical section and two horizontal sections, as shown in Fig. 3.

The value of the wave function on the real section of the integration contour \( C \) differs from the value of the function on the real time axis by the quantity \( Be_{B(t)} \) in the index of the exponential function. With allowance for this contribution, the effective Lagrangian has the form

\[
L = L_0 + \mu (dx/dt)^2 - \mathbf{V}(x, t) + E,
\]

using which, we obtain the transmission coefficient expression

\[
D = \frac{\sigma}{\pi} \frac{\partial^2}{\partial x^2} \int_0^\infty e^{-\frac{1}{2} (x^2 + t^2)} e^{i \mathbf{V}(x) + \mathbf{V}(E) + \int_0^t \mathbf{V}(x) dx} dt,
\]

where \( D \) is the area computed exactly with the formulae (7) and (8). Here it is important that the energy density of the action \( \mathbf{A}(E) \) is equal to \( -2\pi \sigma t \), which was the case in the absence of the variable perturbation. The point is that, because the presence of the action is an external quantity, the term with \( E \) in the expression (8) makes a contribution to \( \delta \ell/\delta \mathbf{V} \), and \( \sigma \) is the distance between the real ends to the contours \( C \). Of course the quantity \( \sigma \) may itself depend on the amplitude of the phase \( \mathbf{V} \). Thus, in the case of a dynamic equilibrium the barrier penetrator factor is found by substituting into (8) the real trajectory satisfying the condition

\[
\tau = t_0/2,
\]

which implicitly selects the energy of the particles tunnelling through the barrier.

The scope of the general expressions obtained in the present section is revealed below in a number of specific examples.
The depth of the potential well in this case is small compared to the overall scale of the potential, and the shape of the potential in the vicinity of the barrier is well described in the tunneling approximation, which is quite adequate for the computation of $\alpha_0$, but which, as has already been stated, leads to the result that $\tau_0 = 0$. To find the finite $\tau_0$ value, we must remember that in fact the potential energy of a Josephson junction is described by an oblique sinusoid, along which the particle slides for an infinitely long time. By turning the integration contour in the expression (11) in the direction of the imaginary $x$ axis we shall be able to make it possible for the particle to reach the point where $V(x) = \infty$, but now complex, time, so that $\tau = 1.177 \omega_0^{-1}$, where $\omega_0$ is the Josephson plasma frequency, i.e., the frequency of very small oscillations of the particle in one of the troughs of the sinusoidal potential when it is not slipping. Let us emphasize that $\tau_0$ tends to a finite value as the direct current approaches the critical value, whereas the subharmonic motion time $\tau_0$ tends in this case to infinity. This fact sharply distinguishes tunneling in Josephson junctions from the examples considered in the previous paper, and demonstrates the importance of the analytic structure of the potential as a whole, i.e., in the regions far from the barrier region.

The exponential field enhancement has been noted by Vayrynen and Rynne in the case of electron tunneling between $\delta$-function wells, when $\tau_0$ is equal to the time of flight across the forbidden region, and by Suslov in the case of tunneling through a triangular potential barrier.

Let us compute the energy distribution for the particles that have tunneled through. The time dependence of the wave function can be explicitly found from the solution to the Hamilton-Jacobi equation. In the first part of the limit $\hbar \omega_0 x$ we have

$$P(x) = \left[ \frac{\hbar}{i} \left( \frac{\partial}{\partial x} \right) \psi(x) \right]^2$$

from which we can find the answers for potentials of specific forms.

Notice that the tunneling problem in a high-frequency field $(\omega \gg \hbar \omega_0)$ cannot be solved with the aid of Kapitza's quasistatic method, since in complex time the field is not an oscillating one, and the amplitude of the alternating field in the region of interest is much smaller than the intensity of the constant field.

6. TUNNELING DECAY OF A METASTABLE STATE IN A WEAK ALTERNATING FIELD

In the preceding section we investigated the effect of an alternating field on the transparency of an isolated potential barrier. If the initial state corresponds to a particle located in a potential well, the particle motion in zero field is finite and periodic: $x(t + T) = x(t)$, where $T$ is the period of the particle vibrations in the well. Taking this into account, we can transform the contour $C_{\omega} = C + \gamma$ in Fig. 2 into a series of closed contours differing from each other by a $\alpha_0$ shift along the real $t$ axis. The function $x(t)$ has the same form on all contours; therefore, the summation over the contours corresponds to our going over from (9) to the following expression:

$$A_r = \frac{\omega_0}{\Omega} \int \frac{d \Omega}{2\pi} \left[ \frac{d}{\partial \Omega} \right]^{2} \hat{\Psi}(x, \Omega) \right]_{\Gamma}$$

where the integration is along the contour shown in Fig. 1.

The reflection coefficient is given by the ratio of the amplitudes of the incident and reflected waves. Let the particle be incident on the barrier from the left. Then the classical trajectory $x(t)$ describing the reflection of the particle from the barrier is specified in complex plane by the contour $C_r$ in Fig. 4. The section $1 \rightarrow 2$ corresponds to the incident particle; the section $3 \rightarrow 4$, to the reflected particle, and the section $4 \rightarrow 3$, to the transmitted particle. At the remote ends of the sections 1 and 3 we have $\omega = \infty$. The section 2 contains the turning point, where $dx/d\Omega = 0$ and the coordinate $x$ is complex, in accordance with the fact that the turning point does not exist in classical mechanics. With allowance for the foregoing

$R(\Omega) = R(0) \exp \left[ - \frac{\omega_0}{\Omega} \right] \int \frac{d \Omega}{2\pi} \left[ \frac{d}{\partial \Omega} \right]^{2} \hat{\Psi}(x, \Omega) \right]_{\Gamma}$

As before, in the high-frequency limit the dominant contribution to the integral is made by the potential's singular points, the behavior of $x(t)$ in the vicinity of which has already been investigated. It is clear that, for potentials with power-law singularities, the correction to the action will be, as before, given by the expression (13), with the only difference that we should now use for the imaginary part of the time of motion to the singular point $x$, the expression

$$\tau = 1 \int \frac{\left( \frac{d}{\partial \Omega} \right)^{2} \hat{\Psi}(x, \Omega) \right]}{\Omega^{2}}$$

where $\omega = \infty$ is an arbitrary real coordinate.

For the particular case of a potential of the form $V(x) = \theta \cos \omega x/\alpha_0$, we have

$\psi(x) = \sum_{n=0}^{\infty} \frac{1}{\alpha_0} \left[ \frac{\left( \frac{1}{\omega} \right)}{\left( \frac{1}{\omega} \right)} \right]^{n} \left( \frac{\sin \omega x}{\omega} \right)$

from which we can obtain by taking account of the nonlinear effects or the damping of the oscillations. The complete solution to the problem of the decay of a metastable state in an oblique sinusoidal potential and in a weak alternating field is given in Ref. 11.

6. ABOVE-BARRIER REFLECTION IN A WEAK ALTERNATING FIELD

In the static situation the coefficient for above-barrier reflection is given by the expression

$$R(0) = \exp \left[ - \alpha_0 \right] \int \frac{d x}{\alpha_0} \left( \frac{d}{\partial \Omega} \right)^{2} \hat{\Psi}(x, \Omega) \right]_{\Gamma}$$

Where $x_s$ is the real and $\alpha_0$ the complex root of the equation $V(x) = \delta x_0$ indicating the position of the turning point. The correction due to the alternating field is given, as before, by the formula (9), but the integration contour will now be different.
Let us note that the equality of $\tau_1$ and Im $\tau_2$ is due to the special choice of potential, and does not hold in the general case.

From the mathematical standpoint the above-barrier reflection effect is similar to the phenomenon of nonconservation of adiabatic invariants. The results of the present section show that a weak variable perturbation should anomalously greatly change the adiabatic invariance if the frequency of the perturbation exceeds the characteristic reciprocal time of the variation of the system's parameters.

7. EXACT SOLUTION FOR A TRIANGULAR BARRIER: FIELD EMISsION IN AN ALTERNATING FIELD

In the preceding sections we computed the corrections to the argument of the tunneling exponential function that are linear in the alternating field. The exact solution in the nonlinearity case is possible only for potentials of a specific form. Below we consider in detail the practically important case of the triangular barrier, a case which has a direct bearing on field emission. We shall assume that, as shown in Fig. 1, a particle of energy $E$ strikes a barrier of height $V$ from the left, and that a constant, $V_0$, and an alternating, $V cos \Omega t$, field exist beyond the barrier. The assumption that there is no electric field in the region to the left of the barrier not only simplifies the problem, but also corresponds to the realizable conditions in which the field is screened off in the vicinity of the metal surface. These assumptions reduce the problem to the problem of computing the imaginary part of the action

$$S = \int \left[ \frac{m}{2} \left( \frac{dz}{dt} \right)^2 + V + Wz + c os \Omega t \right] dt,$$

where $\tau_1$ and $\tau_2$ are the respective instants at which the particle goes, and emerges from, under the barrier. The trajectory $z(t)$ can be found from the equation

$$m \frac{d^2 z}{dt^2} = -V + Wz + c os \Omega t,$$

with the boundary conditions

$$z(0) = 0, \quad \left( \frac{dz}{dt} \right)_{t=0} = (12(V-E)/m)^{1/2}.$$

As stated in Sec. 3, to compute the minimum value of the imaginary part of the action, it is sufficient to limit ourselves to real trajectories. It is easy to verify that the real solution to Eq. (17) corresponds to the situation in which the particle emerges from under the barrier at the instant when the alternating field has its maximum intensity. For definiteness, we shall therefore assume that $\tau_2 = 0$. The solution of (17) with the boundary condition $(dz/dt)_{t=0} = 0$ allows us to find $\tau_1$ from the condition ($18$), the quantity $\tau_1$ is turning out to be purely imaginary. The real trajectory satisfying the indicated conditions has the form

$$x(t) = (V + E)/2m + (\Delta \Omega \cong \Omega) \frac{\Omega}{m^2},$$

where $\tau$ varies along the imaginary axis and the quantity $\tau$ can be found from the equation

$$\frac{d\tau}{d\Omega} = x,$$

which corresponds to a multiquantum non-tunneling penetration of the barrier, when we have for the transmission coefficient

$$D e^{-\frac{1}{\hbar}(2\Delta \Omega x)^{1/2}}.$$
24 and 25, where the constant and alternating fields are considered separately. For a one-dimensional two-band semiconductor with a spectrum given by

\[ \nu(q) = (qz/2) + (2\pi/qz) \]

the Lagrangian that takes account of the alternating, \( \nu \) with \( \nu_0 \), and constant, \( \nu_0 \), fields has the form

\[ L = \nu_0/2 - \frac{1}{2} \left( \frac{d\nu}{dt} \right)^2 - \frac{1}{2} \left( \frac{d\nu_0}{dt} \right)^2 \]

The classical trajectory can be found in its explicit form:

\[ \frac{dz}{dt} = \left( \frac{\nu(q) + \nu_0}{\nu_0} \right) \sin \phi \]

\[ \frac{d\phi}{dt} = \frac{1}{\nu_0} \left( \frac{d\nu}{dt} \right) \sin \phi \]

The tunneling transition probability in the semiclassical limit can be written in the form (32), that is obtained with the Lagrangian (34), using \( C_1 \) and \( C_2 \), the contours in Fig. 7.

The square root in (35) has opposite signs on opposite banks of the branch cut, and the integral continues are the zeros of the term \( q \) corresponding to the zero of the radicand in (35). As in the preceding section, the instant at which the particle emerges from the forbidden region should correspond to the maximum value of the field. The argument of the tunneling exponential function then has the form

\[ A = \frac{\sqrt{2}}{\nu_0} \int \left( \frac{q z}{2} \right)^{1/2} \left( \frac{q z}{2} \right)^{1/2} \]

where the upper integration limit corresponds to the root of the integration. Let us introduce the function \( v = \sqrt{2} \nu_0 \), which is the time required by the particle to gain in the static field \( \nu_0 \), an energy of the order of the forbidden band width \( \nu_0 \), and rewrite the expression (36) in the form

\[ A = \int \left[ \frac{q z}{2} \right]^{1/2} \left( \frac{q z}{2} \right)^{1/2} \]

where

\[ \frac{\nu_0}{\sqrt{2}} \frac{d\phi}{dt} \]

is the action in the absence of an alternating field and \( \nu_0 \) in the root of the equation

\[ \nu_0 \left( \frac{d\phi}{dt} \right) = \nu_0 \]

\[ \nu_0 \left( \frac{d\phi}{dt} \right) = \nu_0 \]

which corresponds to a quantum transition not occurring in the forbidden band under conditions when the effect of the constant field can be ignored. Further increase of the field amplitude \( \nu_0 \) leads to the situation in which the quantity \( v \nu_0 \), and the action is given by the expression

\[ A = \left( 2 \nu_0^2 \right)^{1/2} \left( \frac{q z}{2} \right)^{1/2} \nu_0 \]

where the effect of the constant field can be ignored if

\[ \left( \frac{d\nu_0}{dt} \right)^2 < \frac{1}{2} \left( \frac{d\nu}{dt} \right)^2 \]

in the region of relatively small \( \nu_0 \) the expression (47) goes over into (46), while for \( \nu_0 \gg \nu \) the tunneling process is quasistatic, and

\[ A = \nu_0 \left( \nu_0 \right)^{1/2} \]

which is entirely similar to the expression (33) in the preceding section.

We shall not enumerate the tunneling regimes, since the corresponding field intensity and frequency regions coincide exactly with the regions given in the preceding section, and the entire difference amounts to some change in the dependences of the action from the indicated parameters.

9. CONCLUSION

Our purpose in the present paper was to investigate in detail the effect of an alternating field on semiclassical processes. The most striking effect here consists in the exponential enhancement of the high-frequency field during the motion of the particle in imaginary time. We have found that, in the particular cases of field emission and interband tunneling, the argument of the tunneling exponential function depends non-linearly on the ratio \( \nu_0/\nu \), while the latter is of the order of unity. The relative correction to the action in the case of the same order of smallness as the parameter \( \nu_0/\nu \), but the absolute value of the correction should be large, if the semiclassical approximation is to be applicable. It is, apparently, in this regime that the qualitative difference between the variable and constant fields is most clearly manifested. As the field intensity is increased further, the dependence of the action on the field amplitude \( \nu_0 \) becomes much weaker, specifically, it becomes logarithmic. We have computed these dependences in the entire domains of the parameters, but the conditions of applicability of the corresponding analytic expressions are too rigid to be fulfilled in the case of real systems. Thus, we can hope for the fulfillment of the condition \( \nu_0 \geq 1 \) but, when the condition \( \nu_0 \geq 1 \) is fulfilled simultaneously with the condition \( \nu_0 \geq 1 \), the decay probability expression \( -\alpha \nu_0/\nu \) (where \( \alpha \) is the order of unity) will be so small that the decay will be unobservable.

In the case of charge exchange between deep-lying centers in semiconductors, when the distance between them is large, typical values of the action \( \nu_0 \left( \frac{d\phi}{dt} \right)^2 \) are of the order of 10 to 15, as is possible in tunneling chemical reactions, or in field emission, then the exponential enhancement of the alternating field can be observed in the case of the absorption of a small number of quanta, when the quantity \( \nu_0 \left( \frac{d\phi}{dt} \right)^2 \geq 1 \). For the triangular barrier

\[ \frac{d\phi}{dt} = \frac{1}{\nu_0} \left( \frac{d\nu_0}{dt} \right)^2 \exp \left[ -A \left( \nu_0/\nu \right) \right] \]

which corresponds to perturbation theory in the case of high frequencies, when the semiclassical approximation is inapplicable. Nevertheless, as can be seen from the expression given above, the effective field turns out to be exponentially enhanced in this limit as well.