Cyclotron resonance in multivalley semiconductors with dislocations

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The splitting of Landau levels by dislocations in a multivalley semiconductor is analyzed. At cyclotron resonance in a magnetic field parallel to the dislocations, satellites appear at a distance $\beta \delta$, from the fundamental level, where $\beta = 1$. The broadening of a satellite line when the magnetic field deviates slightly from the direction of the dislocations is studied.

1. INTRODUCTION

The customary approach in the analysis of interactions between current carriers in an uncharged dislocation is the strain-energy approximation. In a multivalley semiconductor, a dislocation also produces an effective vector potential for the electrons of each valley. For example, for a cubic crystal, in which the valley vector $k$ lies along an axis of symmetry no lower than twofold, we have

$$
A_x = \alpha_0 \delta_0 \delta_2 \frac{\partial f}{\partial \gamma_2},
A_y = \alpha_0 \delta_0 \delta_2 \frac{\partial f}{\partial \gamma_1},
A_z = \alpha_0 \delta_0 \delta_2 \frac{\partial f}{\partial \gamma_3},
$$

where $\alpha_0 = \alpha_0/\alpha_2$, is the distortion tensor, and $\gamma_i = 1$. The first term here is purely geometric, while the three others result from a displacement from the valley vector $k$ space because of the deformation. In the most common case, $k = K/2$ ($K$ is a reciprocal-lattice vector), there are no deformation terms, and the dislocation is equivalent to a narrow solenoid with a flux equal to half the quantum. In this case, the magnetic field is parallel to the dislocation. In this study, we discuss the effect of the magnetic field on the dislocation of a Landau level and parallel to the $x$ plane. In dimensionless coordinates, this angle is related to the real angle $\beta$ by

$$
\cos \beta = \frac{(\cos \theta - \cos \phi)}{(\sin \theta)^2},
$$

where $\gamma^x$ is the magnetic field vector, and $\gamma^x$ is a generalized Laguerre polynomial. The eigenvalues are (in dimensionless units)

$$
E = \frac{\hbar^2 m_0}{2} \left( \frac{n^2}{2m_0} \right),
$$

where $\hbar$ is the reduced Planck constant, and $n$ is the density of states. Small radius would be a small perturbation in the presence of the vector potential (1), since all the wave functions decay in power-law fashion as $r \rightarrow 0$.

In the presence of a dislocation, the Hamiltonian of an electron in a magnetic field is

$$
H = \sum \left( \frac{1}{2m_0} \left( \mathbf{p} - \mathbf{A} \times \mathbf{r} \right)^2 - \frac{\hbar^2}{2m_0} \right) + \frac{\hbar^2}{2m_0} \left( \mathbf{p} - \mathbf{A} \times \mathbf{r} \right)^2 + U (r).
$$

The subscripts $1$ and $2$ specify the components respectively parallel and perpendicular to the vector of the bottom of the valley, $\mathbf{A}(x,y, z)$ is the strain energy, and $A^2 = (1/2) \left[ \epsilon \mathbf{H} \right]^2 / 2$.

2. SPLITTING OF A CYCLOTRON-RESONANCE LINE IN A MAGNETIC FIELD PARALLEL TO A DISLOCATION

We begin with the very simple case in which there is only a topological interaction between the electron and the dislocation. The line splitting is described by the expression for $A^2$, and the magnetic field is parallel to the dislocation. In this case, the dislocation is equivalent to a solenoid with a flux $\Phi = (2\pi h/2)\alpha$, in units of the quantum of flux (h is the Planck constant). We introduce the dimensionless units

$$
\alpha^2 = \frac{m_0}{\hbar^2} \frac{\partial f}{\partial \gamma_2},
\alpha^3 = \frac{m_0}{\hbar^2} \frac{\partial^2 f}{\partial \gamma_1 \partial \gamma_2},
\alpha^4 = \frac{m_0}{\hbar^2} \frac{\partial^3 f}{\partial \gamma_1^3},
$$

Here

$$
\alpha^1 = \frac{m_0}{\hbar^2} \frac{\partial f}{\partial \gamma_2},
\alpha^2 = \frac{m_0}{\hbar^2} \frac{\partial^2 f}{\partial \gamma_1 \partial \gamma_2},
\alpha^3 = \frac{m_0}{\hbar^2} \frac{\partial^3 f}{\partial \gamma_1 \partial \gamma_2},
\alpha^4 = \frac{m_0}{\hbar^2} \frac{\partial^4 f}{\partial \gamma_1^4}.
$$

The solutions of this equation is

$$
\psi_{n\alpha} (r, \theta) = \frac{1}{2} \left( 1 + j \phi \right) \exp \left( \frac{j \alpha \Phi}{2} \right).
$$

The perturbation lifts the degeneracy at the Landau level for the given $\alpha$. In the absence of a magnetic field, the solutions which we have chosen here, the states with $p_x + \alpha y = \alpha$ have an identical energy, i.e., there is no degeneracy for the given $\alpha$. We write the eigenfunctions as expansions in the unperturbed states:

$$
\psi_{n\alpha} (r, \theta) = \sum \psi_{n\alpha} Q_{n\alpha} (r, \theta).
$$

The coefficients $Q_{n\alpha}$ obey a secular equation of infinite order:

$$
\Delta (\alpha, \gamma) = \int_0^{2\pi} d \phi \psi_{n\alpha} (r, \theta) \psi_{n\alpha} (r, \theta) \frac{\partial f}{\partial \gamma_2}.
$$

The intensity of the transition accompanied by the transfer of an energy $\alpha \phi$ is determined by the square of the matrix element:

$$
M^{(\alpha)} (\gamma) = \int_0^{2\pi} d \phi \psi_{n\alpha} (r, \theta) \psi_{n\alpha} (r, \theta) \frac{\partial f}{\partial \gamma_2}.
$$

Since the matrix element of the dipole-moment operator is nonvanishing only for the states with $\alpha = \pm 1$, we can put $M^{(\alpha)} (\gamma) = 0$.

3. UNSHAPED IN A SMALL ANGLE BETWEEN THE MAGNETIC FIELD AND THE DISLOCATION

The line shape is sensitive to a deviation of the angle $\alpha$ from the direction of the dislocation. Let us assume that the dislocation makes a small angle $\beta$ with the direction of the magnetic field and is parallel to the $x$ plane. In dimensionless coordinates, this angle is related to the real angle $\beta$ by

$$
\cos \beta = \frac{(\cos \theta - \cos \phi)}{(\sin \theta)^2}.
$$

In terms of these new variables, the Hamiltonian becomes

$$
\mathbf{H} = \frac{1}{2} \left( \mathbf{p}^2 + \frac{\hbar^2}{m_0} \right) + U (r).
$$

Small deviation of the magnetic field from the direction of the dislocation thus gives rise to a perturbation

$$
\Delta (\alpha, \gamma) = \frac{1}{2} \left( \mathbf{p}^2 + \frac{\hbar^2}{m_0} \right) + U (r).
$$

How does this perturbation affect a transition between the states $\alpha = \pm 1$, for $m = 0$? It is a single degenerate level with an energy $E = E_{n\alpha} (1/2 - \alpha)$ ($\alpha = 0$).

Since the Hamiltonian does not contain $z$, the eigenstates can be chosen in the form

$$
\psi_{n\alpha} (r, \theta) = \exp (\alpha \phi) \phi (r, \theta).
$$

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$$

Accordingly, if $m$ and $\alpha$ in Eq. (3) varied from $m = 0$ to $m = 1$, the solution of this equation would be an eigenvector of the operator $\mathbf{P}$ and would have $\mathbf{P} \psi_{n\alpha} = \alpha \psi_{n\alpha}$. Since the relation $m = m' \psi_{n\alpha}$ holds in Eq. (3), we can choose the solution of this equation in the form of a linear combination of eigenvectors of the operator $\mathbf{P}$ which satisfies the conditions $C_{m'\alpha} = 0$ and $C_{m\alpha} = 1$. The eigenvectors of the operator $\mathbf{P}$ can be found from

$$
\mathbf{P} \psi_{n\alpha} = \alpha \psi_{n\alpha}.
$$

This equation has two linearly independent solutions:
where $e^* = e_p = e_e = e = (2 + p^2)^{1/2}$. The coefficients in the linear combination are chosen in such a way that conditions $C_{11}^* = C_{22}^* = 0$. Each eigenvalue $\xi$ is thus doubly degenerate. We can choose two orthogonal solutions $C_1^*$ and $C_2^*$.

The lineshape is determined by the combination $[C_1^*]^2 + [C_2^*]^2$, for which the following expression can be derived:

$$[C_1^*]^2 + [C_2^*]^2 = \frac{16\tau q^*}{\Gamma^{(1+\alpha)} e^{2 + (\beta - 1)\alpha} e^{2 + (\beta - 1)\alpha}} \left[ \frac{D_{-1}^*(\frac{1}{2}p^2)\bar{p}^2}{\Gamma^{(1+\alpha)} e^{2 + (\beta - 1)\alpha} e^{2 + (\beta - 1)\alpha}} \right]$$

In the limiting case $\alpha \rightarrow 0$, the expression for $M'(p)$ simplifies:

$$M'(p) = \frac{\sin \alpha}{\sin \alpha(1 - \alpha)} \Gamma \left( \frac{1}{2} \right) D_{-1}^*(\frac{1}{2}p^2)\bar{p}^2$$

In this case the line remains symmetric with respect to the frequency ($1 - \alpha$), and has a width on the order of $\Delta \nu = \alpha$.

If only the zeroth Landau level is filled in the ground state, the inequality $p < 1$ holds for all the important $p$.

This paper considers the macroscopic consequences of, and the microscopic mechanisms underlying, the following: a current and a magnetic field, a relation which is possible in homogeneous conducting media:

$$\mathbf{P} = \mathbf{M}^* \mathbf{H}$$

Such a relation can be realized only in a non-equilibrium medium. Indeed, the Onsager symmetry relations are valid in the equilibrium case. As shown in the Appendix, a direct consequence of these relations and (1) would be a relation between the magnetic-moment density and the vector potential of the electromagnetic field:

$$\mathbf{M}^* = \mathbf{M}^* \mathbf{H}$$

which is not possible on account of gauge invariance. In superconducting materials gauge invariance is violated, and (1) can be used as a reference point (2).

In the general case in which the vector potential $\mathbf{A}$ is determined by the field $\mathbf{B}$, and $\mathbf{M}^*$ contains the density of the system with respect to the magnetic field $\mathbf{B}$.

In the equilibrium case, it is clear that $\mathbf{A}$ should contain a term $\mathbf{A}$ which is proportional to the gauge field, and $\mathbf{M}^*$ must be such that $\mathbf{M}^* \mathbf{H}$ = 0.

In conclusion, it is shown that a multivalley semiconductor contains uncharged dislocations which affect the maximum of the Brillouin zone, and is not parallel to the dislocation.

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