We are given the truss above and are asked to perform a load analysis of the truss.

The first step to solving problems such as these is to draw a Free Body diagram and to solve for the reaction forces. Before that step, it is beneficial to number each bar and node.

\[ \Sigma F_x = 0 : \quad H_A = 0 \]
\[ \Sigma F_y = 0 : \quad V_A + V_D - 5000 = 0 \]
\[ \Sigma (M_A) = 0 : \quad V_D(30) - 5000(20) = 0 \] (moment taken around pt. A)

Solving for the reactions, we get:

\[ H_A = 0 \text{ lbs} \]
\[ V_D = 3333 \text{ lbs} \]
\[ V_A = 1667 \text{ lbs} \]
Now that the reaction forces are known, we can redraw the truss with the known forces:

- Now we can start applying the method of joints to each node (A → F) to find the forces in the bars.

Let's start with node A:

\[ \theta = \tan^{-1} \frac{5}{10} = 26.6^\circ \quad (\sin 26.6^\circ = 0.45) \quad (\cos 26.6^\circ = 0.89) \]

\[ \begin{align*}
\Sigma F_x &= F_i + F_u \cos \theta = 0 \\
\Sigma F_y &= 1667 - F_u \sin \theta = 0
\end{align*} \]

\[ \begin{align*}
F_u &= 3704 \text{ lbs} \\
F_i &= -3297 \text{ lbs.}
\end{align*} \]

The negative sign on \( F_i \) simply means that the bar is in compression. Note that I had arbitrarily drawn the vector \( F_i \) in tension, but the math tells me that the force is acting in the opposite direction.

We can continue this method for each of the other nodes, now that we know \( F_i \) & \( F_u \).
There is a slightly shorter way of performing this analysis. Let's go back to the FBD:

Note that there are 12 unknowns:
9 internal bar forces
3 external reaction forces

We need 12 equations to solve for them:
6 nodes x 2 degrees of freedom @ each node
= 12 equations

(Basically, we are going to apply the method of joints at each node again, this time considering the reaction forces as unknowns)

Start w/ node A:

\[ \sum F_x = F_1 + 0.89F_4 + H_A = 0 \]
\[ \sum F_y = -0.45F_4 + V_A = 0 \]

Node B:

\[ \sum F_x = -F_1 + F_2 = 0 \]
\[ \sum F_y = -F_5 = 0 \]

Node C:

\[ \sum F_x = F_3 - F_2 - 0.89F_6 = 0 \]
\[ \sum F_y = -0.45F_6 - F_7 = 0 \]
node D:
\[ \Sigma F_x = -F_3 - 0.89 F_8 = 0 \]
\[ \Sigma F_y = -0.45 F_3 + V_0 = 0 \]

node E:
\[ \Sigma F_x = F_9 + 0.89 F_6 - 0.89 F_4 = 0 \]
\[ \Sigma F_y = 0.45 F_4 + F_5 + 0.45 F_6 = 0 \]

node F:
\[ \Sigma F_x = 0.89 F_8 - F_9 = 0 \]
\[ \Sigma F_y = 0.45 F_8 + F_7 = 5000 \]

Now we can put everything into a neat 12x12 matrix:

\[
\begin{bmatrix}
1 & 0 & 0 & 0.89 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.45 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & -0.89 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.45 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & -0.89 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.45 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.89 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0.45 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
F_3 \\
F_4 \\
F_5 \\
F_6 \\
F_7 \\
F_8 \\
U_A \\
V_A \\
V_B \\
V_D \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
5000 \\
\end{bmatrix}
Using a program such as MATLAB or MAPLE, (or even a TI-82 calculator), we find:

\[ F_1 = -3296 \quad 163 \]
\[ F_2 = -3296 \]
\[ F_3 = -6593 \]
\[ F_4 = 3704 \]
\[ F_5 = 0 \]
\[ F_6 = -3704 \]
\[ F_7 = 1667 \]
\[ F_8 = 7407 \]
\[ F_9 = 6593 \]
\[ H_A = 0 \]
\[ V_A = 1667 \]
\[ V_B = 3333 \]

\[ \text{As found before} \]

A couple of things to note:

1. \( F_5 \) carries no load (it is basically useless in the truss).
2. \( F_8 \) carries the most load (we have to set the radius of the bars according to this force).

Also note that \( F_1 = -3296 \) & \( F_4 = 3704 \), the same answers we got by finding the reaction forces first and then applying method of joints.

Both methods work!!!
Now we know \( F_8 \) carries the most load. We also know that all truss members must have the same radius, so we must set our radius so that \( F_8 \) does not fail.

Since the truss must have a factor of safety of 1.5, let's multiply \( F_8 \) by 1.5 and design our truss according to that load:

\[
1.5 F_8 = 1.5(7407) = 11111 \text{ lbs}
\]

To pick a material, we are looking for something that has a high yield stress and low density. (We want to maximize the yield stress/density ratio.)

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield Stress/density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>( 1.96 \times 10^5 )</td>
</tr>
<tr>
<td>Aluminum</td>
<td>( 9.22 \times 10^5 )</td>
</tr>
<tr>
<td>Titanium</td>
<td>( 8.75 \times 10^5 )</td>
</tr>
</tbody>
</table>

Clearly, \textbf{Aluminum} is the material of choice.
Finally, let's solve for the radius & weight.

Stress is defined as:

$$
\sigma = \frac{F}{A}
$$

yield stress \quad cross sectional area

$$
\sigma = 83000 \text{ lb/in}^2
$$

$$
F = 11111 \text{ lb}
$$

$$
A = \pi r^2
$$

$$
\Rightarrow \pi r^2 = \frac{F}{\sigma} \Rightarrow r^2 = \frac{11111}{83000 \pi} \Rightarrow r = \sqrt{\frac{11111}{83000 \pi}}
$$

$$
\Rightarrow r = 0.21 \text{ in}
$$

Let's be careful of units here. Yield stress was given in \text{ lb/in}^2, so the radius should be in inches.

Since the lengths of the truss members are given in feet, let's convert the total length to inches (since \( g \) is given in inches).

**Total length** = \( 10 + 10 + 10 + 10 + 5 + 5 + 11.2 + 11.2 + 11.2 \)

\( = 83.6 \text{ ft} = 1003.2 \text{ inches} \)

**Total weight** = \( 8V = (0.09 \text{ lb/in}^2)(1003.2)(\pi)(0.21)^2 \)

**Total weight** = \( 12.5 \text{ lbs} \)
2. For a propeller of fixed orientation, the twist of the propeller is designed so that each airfoil section is at its optimum angle of attack to the relative airflow. When the forward velocity of the airplane is changed, the angle of attack of each airfoil section changes relative to the local flow direction. In other words, a constant-pitch propeller is operating at maximum efficiency only at its design speed. The solution to this problem is to vary the pitch of the propeller during the flight, so as to operate at near-optimum conditions over the flight range of the airplane.

3. A brief description of the main elements of a turbofan engine:
   - **Diffuser**: Slows down airflow.
   - **Compressor**: Increases the pressure of the airflow by performing work.
   - **Burner**: Mixes the high-pressure air with fuel and produces combustion.
   - **Turbine**: Extracts energy from the flow to drive the shaft, causing a pressure drop.
   - **Nozzle**: Increases velocity to produce thrust, conducts the exhaust gases back to the freestream, and sets the mass flow rate through the engine.
   - **Bypass Fan**: Causes additional air to flow around the engine, thus producing greater thrust and reducing specific fuel consumption. Increases efficiency.

4. Determine the location of the forward-most and rear-most permissible center-of-gravity locations.
   - **Forward-most** $x_{CG}$:
     Sum the moments around the aerodynamic center and set the sum equal to zero.
     \[
     M_{ac} + L_t (10 - 0.1) - W (x_{CG} - 0.1) = 0 \\
     100 - 100 (10 - 0.1) - 1000 (x_{CG} - 0.1) = 0 \\
     x_{CG} = 0.79 \text{ m}
     \]
     This means that the forward limit for the center-of-gravity location is 0.79 m ahead of the wing.
   - **Rear-most** $x_{CG}$:
     The sum of the changes in moments around the center of gravity must be negative. Note that the moment around the aerodynamic center does not change.
     \[
     0 + \Delta L (x_{CG} - 0.1) - \Delta L_t (10 - x_{CG}) < 0 \\
     \Delta L (x_{CG} - 0.1) - (\Delta L/10) (10 - x_{CG}) < 0 \\
     10 x_{CG} - 1 - 10 + x_{CG} < 0 \\
     x_{CG} < 1 \text{ m}
     \]
     The center-of-gravity has to be less than 1 m behind the wing.