ROCKET PERFORMANCE LAB

Objectives
At the end of this problem set, you should be able to:
• Predict the performance of a model rocket under zero-drag conditions
• Explain the effects that drag & wind will have on a rocket’s flight path and performance
• Predict the center of mass and center of pressure locations of a model rocket design
• Assess the stability of a model rocket design
• Explain and follow range safety procedures
• Successfully launch a model rocket

Problem Statement
In this lab, you will work on a small team (3-4 students) to design, analyze, and then flight test a model rocket. The design goal is to build a reliable rocket that will maximize downrange flight distance when launched at an elevation angle that you choose. This requires ensuring that the rocket is stable in flight and will have as little drag as possible.

We will first launch a calibration rocket (made by the TAs) at a 45° elevation angle to factor in the wind effects on a given launch day. Then your team will launch your rocket, and we will assess the distance your rocket flies downrange compared to the calibration rocket. The goal is to be the team with the largest percent increase in downrange distance relative to the calibration rocket.

Your team will receive a set of rocket materials from which you will construct your rocket. You are not allowed to use any additional materials (other than adhesives) or to make any test flights before the official data-collection flight.

Your team will hand in a single report documenting your work. Your grade will depend 50% on the whole team grade, and 50% on your own contribution. We expect each team member to take on an equal load in terms of time and effort. Note that just flying the rocket will not count for much effort.

1. Performance prediction
We will begin by predicting the performance of a model rocket while ignoring the effects of drag, wind, or the mass change as fuel is burned. Assume that the rocket thrust is maintained at a constant elevation angle until burnout, after which the rocket flies ballistically.
Assume the following parameters are known:

- Average rocket motor thrust: \( T \)
- Rocket motor burn time: \( t_b \)
- Total rocket mass: \( m \) (assumed constant)
- Launch elevation angle: \( \theta \) (measured from the horizontal)

(a) Find an equation or set of equations to compute the maximum altitude attained by the rocket. Hint: Do separate computations of just the horizontal components of thrust, acceleration, velocity, and position, and another set of computations for just the vertical components of thrust, etc.

(b) Find an equation or set of equations to compute the total downrange distance flown by the rocket. Same hint as for (a).

(c) The parameters for the calibration rocket are given below. Compute the estimated downrange distance the calibration rocket will fly using your solution from part (b).

\[
\begin{align*}
\text{Average rocket motor thrust: } & T = 3.03 \text{ N} \\
\text{Rocket motor burn time: } & t_b = 0.36 \text{ s} \\
\text{Total rocket mass: } & m = 0.027 \text{ kg} \\
\text{Launch elevation angle: } & \theta = 45^\circ \text{ (measured from the horizontal)}
\end{align*}
\]

2. Concept Development

As a group, brainstorm as many rocket design concepts as you can (varying body length, fin layout, etc.), and then downselect to at least three different likely design concepts. Sketch each of these three designs – these need not be engineering drawings, but just conceptual sketches. Also briefly describe the advantages and disadvantages of each of the three design concepts.

Create a Product Design Matrix that includes all of the design requirements you can identify (see the <LTA.pdf> lecture notes file on the 16.00 web site for an example Product Design Matrix). In the matrix, indicate the importance your team places on each design requirement, and fill out the degree to which each design solution meets each requirement. Using the matrix, select a final design concept.

3. Detailed Design

Carefully measure and weigh the rocket components in your rocket kit. Then, generate a detailed design for your rocket. This should include:

- Engineering drawings (side and top views, dimensioned)
- Computation of the center of mass position (CM), measured from the nose of the rocket
- Computation of the center of pressure position (CP) using the Barrowman method (attached)
- Safety Margin: \( \frac{(CM - CP)}{D} \), where \( D \) is the body diameter.
- Desired launch elevation angle
- Computation of the maximum altitude and downrange distance your rocket will fly, and the expected % increase or decrease relative to the calibration rocket
Your rocket must have a Safety Margin of at least 1, so make and document revisions to your design as necessary.

4. Construction
Build the rocket to the specifications in your engineering drawings. After it is built, double-check the center of mass position by balancing the rocket on your finger or a small dowel and comparing against your estimate from part 3. If the CM is not where you predicted it would be, recompute the Safety Margin using its new position, and make and document any necessary design modifications to ensure the Safety Margin is at least 1.

5. Launch
A sign-up sheet of flight times will be posted for dates between March 7 – March 13. Flights will take place on Briggs Field. You must bring your documentation for the center of mass and center of pressure positions to the field at flight time in order to be allowed to fly. You must also follow all range safety procedures. During the flight, the TAs will measure the actual downrange distance your rocket flies and compare it to the calibration rocket distance. As in the real world, you only get one chance when you launch a rocket, so re-launches will not be allowed.

6. Final Report
Make a single, final team report that includes the following elements:

- Team member names and each member’s specific contribution to the project
- Performance analysis from parts 1(a) – 1(c)
- Design concepts, sketches, and Product Design Matrix
- Detailed design, including all elements listed in part 3
- Any notes on modifications made to the design during or after construction
- A brief flight report, documenting downrange distance flown and comparing this to the calibration rocket and to your predictions. Discuss what contributed to any discrepancies. Also describe and discuss the overall performance of your rocket, whether it met (or didn’t meet) your expectations, failed, etc.

This report is due in class on Tuesday, March 19.
Rocket Construction Tips

As a team, pick up your rocket kit from the TAs in the Gelb lab. Carefully measure and weigh each component so that you can design and analyze your rocket before actually building it. You may want to shorten the body tube and will make your own fins from balsa sheet. You can predict the mass of the fins or body by first determining each part’s mass/length or mass/area and then multiplying by the designed length or area. You only get one chance to build the rocket from the supplied materials, so be sure to carefully plan out your design before construction.

Fins will be strongest when the wood grain runs parallel to the leading edge of the fin. Lightly sanding the rocket body will make the fin attachments stronger when adhesive is applied.

The rocket is launched on a launcher which consists of a steel rod inclined at your chosen launch elevation angle. A short (1-2” long) plastic launch guide tube (a piece of a drinking straw) must be attached to the side of your rocket. Attach the tube starting approximately 2” from the bottom of the rocket, and ensure that the tube is parallel with the rocket body. Inclining the guide tube or fins will cause your rocket to spin about its long axis – this will decrease the energy that is applied toward the rocket’s forward kinetic energy, reducing its flight distance.

The rocket motors have a diameter less than that of the body tube. The rocket motor should first be mounted inside a smaller short tube that exactly fits the motor (provided in your kit), and then design a mount using balsa strips to hold the motor + tube inside the body tube. Design this mount so that the motor assembly fits tightly in the body without any adhesive, which will let you remove the used motor after your launch so the rocket can be reused if you’d like. Because the motor is not glued in place, you will also need to make and glue some form of stop inside the main rocket body to prevent the rocket motor from moving or pushing up inside the body during launch.
**ESTES 1/ 2A3T**

**CERTIFIED VALUES**
- **Total Impulse:** 1.25 newton-seconds
- **Delays:** 2, 4 seconds
- **Propellant Type:** Black Powder
- **Propellant Mass:** 2.0 grams
- **Casing Dimensions:** 13 mm × 45 mm
- **Certification Date:** Continuing
- **Contest Use Date:** Continuing
- **Certification Type:** Model Rocket

**STATIC TEST DATA**
- **Date Tested:** 95-June-24
- **Total Impulse:** 1.09 newton-seconds (σ 0.04)
- **Peak Thrust:** 7.62 newtons (σ 1.17)
- **Burn Time:** 0.36 seconds (σ 0.36)
- **Average Thrust:** 3.03 newtons
- **Mass After Firing:** 3.5 grams

<table>
<thead>
<tr>
<th>Delay Time</th>
<th>Average Measured Delay</th>
<th>Initial Mass</th>
<th>Mfg Recommended Max Liftoff Weight</th>
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</thead>
<tbody>
<tr>
<td>2</td>
<td>1.38</td>
<td>6.4 g</td>
<td>56.6 g</td>
</tr>
<tr>
<td>4</td>
<td>3.08</td>
<td>6.8 g</td>
<td>28.3 g</td>
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</tbody>
</table>

**TYPICAL THRUST-TIME CURVE**

**REMARKS**
- Updated: 8/96

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The total impulse, peak thrust, average thrust and burn time are the same as the averaged static test data on the NAR web site in the certification file. The curve drawn with these data points is as close to the certification curve as can be with such a limited number of points (32) allowed with wRASP up to v1.6.

<table>
<thead>
<tr>
<th>Time (Sec)</th>
<th>Thrust (N)</th>
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<tbody>
<tr>
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<td>1.454</td>
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<tr>
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<td>3.009</td>
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<tr>
<td>0.076</td>
<td>4.062</td>
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<tr>
<td>0.093</td>
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<td>6.068</td>
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<td>0.112</td>
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<td>0.651</td>
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<tr>
<td>0.360</td>
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</table>
A stable model rocket will point its nose into the relative airflow. Model rockets are stabilized aerodynamically.

A stable model rocket must have its Center of Gravity (CG) ahead of its Center of Pressure (CP).

The CG is the balance point of the rocket. The model rotates about this point while in free flight.

The CP is the point where the net aerodynamic force acts on the model.

The Stability Margin (SM) is the distance between the CG and CP (a positive value for a stable rocket). For a safe flight, the SM should be at least as great as the diameter of the widest tube in the model; this is referred to as “one caliber” stability.

The net aerodynamic force, which acts through the CP, produces a torque that acts to turn the nose cone back into the relative airflow.

On an unstable model, the aerodynamic force will tend to push the nose of the rocket further off to the side. This causes the model to thrash wildly around the sky while the motor is thrusting.

An unstable model rocket can be made stable by...
Center of Gravity (Center of Mass) (Balance Point)

The center of gravity of a set of masses arranged along a line can be calculated from the equation:

\[ X_{CG} = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3 + \ldots}{m_1 + m_2 + m_3 + \ldots} \]

Example:

\[ X_1 = 2 \text{ cm} \]
\[ X_2 = 4 \text{ cm} \]
\[ X_3 = 12 \text{ cm} \]

\[ X_{CG} = \frac{4 \cdot (2) + 1 \cdot (4) + 8 \cdot (12)}{4 + 1 + 8} = \frac{8 + 4 + 96}{13} = \frac{108}{13} \text{ cm from ref. line} \]

The choice of where to put the reference line is arbitrary. Any reference line can be used (distances left of the line are negative).

\[ X_{CG} = \frac{(4)(-3) + (1)(-1) + (8)(7)}{4 + 1 + 8} = \frac{-12 + (-1) + 56}{13} = \frac{43}{13} = 3.3 \text{ cm from new ref. line} \]

The CG of an axial-symmetric model rocket can be found using the same equation:

But if you have the completed model you can simply find the balance point.
Center of Pressure
(Barrowman Method)

Normal Force of each component:
\( (C_N)_{N, \text{comp}} \)

Center of Pressure of each component:
\( \bar{X}_{\text{comp}} \)

Nose Tip Reference Line

\( d = \text{reference diameter (base of nose)} \)

\( D = \text{maximum body tube diameter} \)

Overview and definitions
The Center of Pressure (CP) is the point where the net aerodynamic force acts

C.P. positions of components.
Example: \( \bar{X}_{\text{S}} = X_{\text{S}} + \Delta X_{\text{S}} \)

The Static Margin of stability (SM) is the distance from the Center of Gravity (CG) to the Center of Pressure (CP). To insure a stable flight, the CG should be ahead of the CP with a Static Margin equal to, or greater than, the distance \( D \), where \( D \) is the largest body diameter in the model.

Total Normal Force:
\( C_N = (C_N)_n + (C_N)_s + (C_N)_b + (C_N)_f \)

Center of Pressure of Entire Rocket:
\[ \bar{X} = \frac{ (C_N)_n \bar{X}_n + (C_N)_s \bar{X}_s + (C_N)_b \bar{X}_b + (C_N)_f \bar{X}_f }{ C_N } \]

Eliminate or duplicate the shoulder, boattail, and fin terms in the above equations as needed for the configuration of the rocket. Use the same unit on all distance measurements. The resulting CP position (from the nose reference line) will be in the same distance unit.
Nose

Normal Force: \((C_N)_n = 2\) for all shapes.

CP Position:

\[
\bar{x}_n = \begin{cases} 
\frac{2}{3} L & \text{Cone} \\
\frac{1}{3} L & \text{Ellipsoid} \\
\frac{1}{2} L & \text{Paraboloid} 
\end{cases}
\]

Odd-shaped noses can be approximated by drawing an "outline cone" that just encloses the nose. The CP position of the nose is found by:

\[
\bar{x}_n = \frac{2}{3} (L + \Delta L) - \Delta L
\]

All other position measurements on the model are measured from the nose reference shown in the normal manner.

Conical Shoulder

Normal Force:

\[
(C_N)_s = 2 \left[ \left( \frac{d_2}{d} \right)^2 - \left( \frac{d_1}{d} \right)^2 \right]
\]

where \(d\) = base diameter of the nose

CP Position:

\[
\Delta x_s = \frac{1}{3} \left[ \frac{1}{1 - \left( \frac{d_1}{d} \right)^2} \right]
\]

\[
\bar{x}_s = x_s + \Delta x_s
\]

Conical Boattail

Normal Force:

\[
(C_N)_b = 2 \left[ \left( \frac{d_2}{d} \right)^2 - \left( \frac{d_1}{d} \right)^2 \right]
\]

where \(d\) = base diameter of the nose

CP Position:

\[
\Delta x_b = \frac{L}{3} \left[ \frac{1}{1 - \left( \frac{d_1}{d} \right)^2} \right]
\]

\[
\bar{x}_b = x_b + \Delta x_b
\]

Note that \((C_N)_b\) will be negative.
Apply this to each set of identical fins on the model. The fin shape must be approximated by a trapezoid, making sure the area of the trapezoid closely matches that of the original fin.

It is better to err on the side of using a trapezoid that slightly underestimates the area of the actual fin. This will give a conservative estimate of the stability margin of the rocket.

**Elliptical Fin Set**

Normal Force:

\[
(C_N)_f = \left(1 + \frac{f R}{s + R}\right) \frac{4 n (\frac{s}{d})^2}{1 + \sqrt{1 + \left(\frac{2 \lambda}{a + b}\right)^2}}
\]

where:
- \(n\) = the number of fins in the set
- \(d\) = base diameter of nose
- \(f = 1.0\) for 3 or 4 fins
- \(f = 0.5\) for 6 fins

CP Position:

\[
\Delta X_f = \frac{m (a + 2b)}{3 (a + b)} + \frac{1}{6} \left(a + b - \frac{ab}{a + b}\right)
\]

\[
\bar{X}_f = X_f + \Delta X_f
\]

**Normal Force:**

\[
(C_N)_f = \left(1 + \frac{f R}{s + R}\right) \frac{4 n (\frac{s}{d})^2}{1 + \sqrt{1 + 1.623 \left(\frac{s}{a}\right)^2}}
\]

**CP Position:**

\[
\bar{X}_f = X_f + 0.288 a
\]

**References**

Calculating the Center of Pressure of a Model Rocket, Centuri Technical Report TIR-33, by James Barrowman.


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