18.06, Spring 2003

Assignment 8 due Wednesday April 23

To obtain full credit explain your answer and write up all the intermediate steps. We want precise, concise, correctly argued answers.

1. For the following two matrices find an orthogonal matrix that diagonalizes them. (Recall that an orthogonal matrix is a matrix whose columns form a sequence of orthonormal vectors.)

\[
\begin{pmatrix}
1 & 2 \\
2 & -1
\end{pmatrix}, \begin{pmatrix}
8 & -4 & 4 \\
-4 & 8 & 4 \\
4 & 4 & 8
\end{pmatrix}.
\]

Explain how you found \(Q\) (as usual).

2. Let \(f(x, y, z) = x^2 - 3xy + 4z^2 - xz\). Find a symmetric matrix \(A\) such that \(f\) is the following matrix product:

\[
f(x, y, z) = (x, y, z)A \begin{pmatrix} x \\ y \\ z \end{pmatrix}.
\]

Is \(A\) positive definite?

3. Let \(A, B, C\) be symmetric \(n \times n\) matrices.

(a) Show that the differences \(B - A\), \(C - A\) and \(C - B\) are symmetric.

(b) Show that if \(B - A\) and \(C - B\) are positive definite, then \(C - A\) is positive definite.

4. For which pairs of real numbers \(a, b\) is the following matrix positive definite?

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & a \\
0 & 1 & 0 & b & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & b & 0 & 1 & 0 \\
a & 0 & 0 & 0 & 1
\end{pmatrix}
\]