18.06, Spring 2003
Assignment 1, due Wednesday Feb 12

Please put stapled homework in your section’s box in 2-106 by 4pm. No late homework will be accepted.

To obtain full credit, explain your answer and write up all the intermediate steps. We want precise, concise, correctly argued answers.

1. Exercise 18 of Problem Set 2.1 from the book.

2. Solve the following system of equations by performing the algorithm of Section 2.2.

\[
\begin{align*}
    x + 2y - z + t &= 2 \\
    2x + 3y - z + t &= 3 \\
    5x - y + z + 2t &= 0 \\
    -x + y + 3z + 2t &= -1
\end{align*}
\]

3. (a) Solve the following system of equations by performing the algorithm of Section 2.3.

\[
\begin{align*}
    -x + 2y - z + t &= 2 \\
    ax + 3y - z + t &= 3 \\
    5x - y + z + t &= 0 \\
    -x + y + z + t &= -b
\end{align*}
\]

(b) Write down the elementary matrices you used in the process.

(c) For which value of $a$ is the system singular?

(d) For the value of $a$ which makes the system singular (found in (c)), is there a value of $b$ for which there are infinitely many solutions? If so, what is it and what are the solutions.

4. Compute the following matrix product

\[
\begin{bmatrix}
    1 & 3 & 2 \\
    0 & 4 & 1 \\
    2 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    4 & 2 & 1 & 0 \\
    1 & 2 & 0 & 3 \\
    0 & 1 & 5 & 2
\end{bmatrix}
\]

In this case, the computation need not be explained.

5. In this exercise $A$ is an $m \times n$ matrix with entries $A_{ij}$ and that $B$ is a $n \times p$ matrix whose entries are $B_{ij}$.

(a) Suppose that $B_{ij} = 1$ for all pairs $(i, j)$. (In other words, all the entries of $B$ are 1.) What is the $(\ell, k)$ entry of the matrix product $AB$? (Find a formula.)

(b) Suppose that all the entries of $A$ and of $B$ are positive. Explain why the entries of $AB$ are positive.