Organizational Beliefs and Managerial Vision

Forthcoming in the Journal of Law, Economics, and Organization

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June 28, 2004

Abstract

Can managers have an impact on their firm that goes beyond their direct actions and decisions? This paper shows that a manager with strong beliefs about the right course of action will attract, through sorting in the labor market, employees with similar beliefs. This alignment of beliefs gives direction to the firm and has important implications for incentives and coordination. The paper then defines vision, in accordance with the management literature, as a strong belief about the right course of action, and shows that it may be optimal to hire managers with such strong beliefs. Vision will be most important when uncertainty is high and actions are difficult to contract on.

1 Introduction

What is the role of a CEO? How do managers influence their companies’ behavior and performance? While economics has done substantial research on managers’ compensation and incentives, it has usually assumed that a CEO’s influence goes solely through her direct actions and decisions. The influence and role of a CEO, however, should be a research question in itself. The answer to this question is important, not only for incentives, but also for organization design and for our understanding of the heterogeneity in firm behavior and performance (Mueller, 1990; McGahan, 1999).

*MIT - Sloan School of Management, Cambridge MA 02142 (email: evds@mit.edu). As always, I learned a lot from John Roberts’ suggestions and critical questions. His support has made an enormous difference to me. This paper also benefited a lot from the numerous suggestions by Ed Lazear and Garth Saloner, the guidance of Bengt Holmstrom, and the useful comments by Bill Barnett, Robert Burgelman, Jennifer Chatman, Kate Dewhurst, Yossi Feinberg, Bob Gibbons, Thomas Hellmann, Jon Levin, Paul Milgrom, Paul Oyer, Julio Rotemberg, Andrea Shepard, Brian Viard, Muhamet Yildiz, Ezra Zuckerman, and the participants in the seminars at Harvard, MIT, Northwestern, Stanford, UBC, University of Chicago, and USC. I finally thank the editor (Oliver Williamson) and the referees for their very positive impact. The usual disclaimer applies. Financial support from the Interuniversitair College voor Managementwetenschappen (ICM/CIM), Stanford Graduate School of Business, and MIT-Sloan are gratefully acknowledged.
While the management literature has discussed extensively the role of the CEO and of managerial vision, the economic literature on these topics (e.g. Rotemberg and Saloner, 1993; 2000), discussed in more detail below, is more limited. The contribution of this paper is to show that a manager can have an important indirect influence on a firm’s behavior and performance. In particular, strong managerial beliefs attract and retain people with similar beliefs, causing an alignment of beliefs within the firm that has important implications for the firm’s behavior and performance.

The model and results. The paper studies a simple model that focuses on the interaction between a manager’s beliefs and those of her employees. In this model, employees can develop new projects and get rents from projects that get implemented and are successful. The implementation decision, however, is in the hands of the employee’s manager. The manager and employee may openly disagree about which type of project (A vs. B) will be successful. This means that I do not impose the common prior assumption, an approach that I will discuss in more detail.

To derive the sorting effect induced by managerial beliefs, we first need to understand the impact of managerial beliefs on the employees’ effort and utility. The paper shows that a stronger belief of the manager increases the effort and utility of employees who agree with her. The reason is that these employees get easier approval for the projects they undertake, which gives them a higher expected return on their efforts. Stronger managerial beliefs will decrease the effort and utility of employees who disagree with the manager. These differential effects on utility then give rise to sorting: a firm attracts employees with beliefs that are similar to those of its manager. The resulting alignment of beliefs gives direction to the firm and improves coordination, since it indirectly aligns the beliefs of different employees. Most important, it eliminates the employees who got most demotivated and the lowest satisfaction from these strong managerial beliefs. This suggests that strong managerial beliefs might be profitable.

In line with the original management literature on the topic, I then define ‘vision’ as a strong belief by the manager about the future and about the right course of action for the firm. To evaluate the profitability of hiring a manager with such vision, I take the perspective of an outsider, such as the board, with a ‘reference’ belief and show that it is often optimal for such outsider to hire a manager whose beliefs are strictly stronger than his own. Such vision will be most effective with high uncertainty and low contractibility. This suggests that vision will be more important for e.g. high-tech industries and start-ups than for established firms in mature industries.

Although this paper concludes that vision may be valuable, casual empiricism and the management literature also suggest that vision has its dark side. Indeed, apart from the fact that strong beliefs lead to ex-post inefficient investment levels and to slow learning in the face of disconfirming evidence, ‘vision’ may become an easy excuse for inefficient managerial overconfidence while boards may hire managers with vision just to hide their own lack of confidence (Khurana, 2002). This dark side of ‘misplaced vision’ should not stop us, however, from exploring its potential benefits. Management studies conclude that vision
is a key characteristic characteristic of effective leaders (Bennis and Nanus, 1985; Donaldson and Lorsch, 1983; The Economist Intelligence Unit and Korn/Ferry International, 1996), while casual empiricism suggests that managers ‘who know what they want’ are often most effective. Systematic studies should help us assess when and to which degree such strong beliefs can be appropriate. To that purpose, this paper abstracts for now from the agency issues mentioned above.

The literature. The idea that a manager’s influence goes beyond her direct actions and decisions is well accepted in the management literature. The literature on culture, for example, has stressed the role of a leader’s values and beliefs in the formation of corporate culture (Donaldson and Lorsch, 1983; Schein, 1985; Kotter and Heskett, 1992). The literature on vision (Bennis and Nanus, 1985; Tichy and Devanna, 1986) also focuses on how the characteristics and beliefs of managers influence employees’ actions and decisions.

Economics has largely neglected these issues. An important exception are Rotemberg and Saloner (2000) who present a formal model of vision. Extending their work on leadership styles and strategy (Rotemberg and Saloner, 1993; 1994), they consider a firm with two employees, or product divisions, working on different projects. Vision in their model is a bias of the manager that makes him favor one division over the other. Such vision improves the incentives of one employee at the cost of reducing the incentives of the other. While their paper shows that managerial preferences may matter to firm performance, it does not allow to address the question of firm heterogeneity: in their world, all firms would hire equally biased managers and perform exactly the same. The focus on within-firm bias also limits the model to incentive considerations and does not allow to study such issues as decision making, sorting or coordination. In more recent work, Hart and Holmstrom (2002) consider how managerial characteristics such as vision may be a determinant of firm boundaries. Their argument is essentially that in a world with incomplete contracts, a manager’s bias matters and that different activities may therefore need different managers. More distantly related is the work by Goel and Thakor (2000), who argue that people who underestimate project risks have a higher probability to win in tournaments and thus get elected as leader, and that such overoptimism may compensate for risk aversion. Hermalin (1998) assumes that leaders have better information and then studies how they can communicate that information to others.

On the empirical side, Bertrand and Schoar (2003) show that a firm’s policies depend on the identity of the CEO. This, and especially their finding that a manager’s policies are correlated with whether he or she attended an MBA program, fits the idea that CEO’s may differ in their beliefs about what is right and that these belief differences have real implications for firm behavior and performance. This also fits the observation that much of the performance differences among firms cannot be accounted for by industry or market share (Schmalensee, 1985; Rumelt, 1991; McGahan and Porter, 1997). In a different strand of literature, Baum, Locke and Kirkpatrick (1998) find evidence of a positive influence of vision on venture growth and survey earlier empirical studies on managerial vision.

On the methodological side, finally, while financial economics has used heterogenous
priors since Harrison and Kreps (1978), other fields have only recently begun to explore its implications (e.g. Morris, 1994; Feinberg, 2000; Yildiz, 2000; Van den Steen, 2001; Fang and Moscarini, 2003).

Relative to the economic literature, the central contribution of this paper is the sorting effect: it considers the formation of a firm’s work force from a perspective other than productive ability, throws a new light on the role of the manager, and derives real economic consequences from this sorting, in terms of incentives and coordination. Relative to the managerial literature, the paper provides a transparent and formal analysis of vision in the sense of strong beliefs, suggests new insights such as the sorting effect, and derives predictions when vision will be most important.

The next section discusses the model and relates my definition of vision to that of the management literature. Section 3 considers the impact of managerial beliefs on employees’ decisions, effort and utility. On that foundation, section 4 derives the sorting effect and discusses its implications for incentives and coordination, while section 5 considers when vision would be profitable. Section 6 discusses issues with testing the theory, while section 7 concludes. Appendix A discusses potential variations and objections, while appendix B contains the proofs.

2 The Model

The model is meant to capture the interaction between a manager and her immediate subordinates. The typical case would be a CEO and her management team. For clarity, however, I will use the terminology ‘manager’ and ‘employee’.

The actions and timing are represented in figure 1. After choosing a firm to work for, an employee can try to develop a new project. The employee first chooses the project type, $A$ or $B$, which are mutually exclusive, and then spends effort $e \in [0, 1]$ to develop the project. Spending effort carries a personal cost $c(e)$. The probability that the employee generates a project equals his effort $e$. If the employee generates a project, the manager

\begin{itemize}
  \item[1] The firm gets $\gamma_M$ from a successful project, 0 from a failure.
  \item[2] The employee gets $\gamma_E$ from a successful project, 0 from a failure.
\end{itemize}

Figure 1: Timeline of game

1 Note, however, that a CEO’s vision may influence employees $n$ levels down the hierarchy via a cascade of the mechanism discussed in this paper.
decides whether to implement it, considering not only the project’s expected revenue but also its implementation cost $I$, which is drawn from a uniform distribution and is observed only after the project has been generated. The firm gets a payoff $\gamma_M$ from a project that was implemented and successful, while the employee gets a payoff $\gamma_E$. Failures generate no payoff for either the firm or the employee.

The key element of the model is the uncertainty about which type of project will be successful. In particular, the success of a project depends on its fit with the (unknown) state of the world $X \in \{A, B\}$, with $X$-type projects being successful if and only if the state is $X$. The state may include any factor that has a bearing on what the optimal action is, including evolution of the industry, core competences of the firm, or ‘the right way of doing things’.

All agents in the model have their own subjective belief about the likelihood of each state. I will use the notation $\mu_{i,Y}$ for the probability that agent $i$ assigns to the event that the state is $Y$. Employee $E$, for example, believes that with probability $\mu_{E,A}$ the state is $A$. Managers, when deciding on implementation, maximize firm profits using their subjective belief $\mu_{M,Y}$. Employees, when choosing projects and spending effort on developing them, maximize their expected revenue net of any cost of effort, using their subjective beliefs $\mu_{E,Y}$.

The agents’ beliefs may differ but are common knowledge. This implies, by Aumann (1976), that the agents start from differing priors. Such heterogeneous priors do not contradict the economic paradigm: while rational agents should use Bayes’ rule to update their prior with new information, nothing is said about those priors themselves, which are primitives of the model. In particular, absent any relevant information agents have no rational basis to agree on a prior. Harsanyi (1967), for example, observed that ‘by the very nature of subjective probabilities, even if two individuals have exactly the same information and are at exactly the same high level of intelligence, they may very well assign different subjective probabilities to the very same events’. The reason for following this approach is simply that common knowledge of differing priors is the most transparent and parsimonious way to study the issues in this paper.\footnote{Note that with differing priors, agents will not update their beliefs merely because they are confronted with someone with a different opinion, unless there is also private information.}

Working with differing priors raises the issue how to measure expected profits and thus how to determine the optimal strength of beliefs. Note first that, when determining ‘optimal’ beliefs and the ‘optimality’ of vision, I abstract from the agency and bounded rationality issues that give rise to a dark side of vision, mentioned in the introduction. To determine then the optimal strength of beliefs in the face of differing priors, I will use the perspective

\[\text{This deterministic connection between state and success simplifies the theory but is not necessary. In particular, in a multi-period model, it might be better to specify a probabilistic connection so that the state does not get completely revealed by the outcome of the first period.}\]

\[\text{For a more in-depth discussion of these issues, see Morris (1995), the discussion between Gul (1998) and Aumann (1998), Yildiz (2000), or Van den Steen (2004a). The last argues explicitly that differing priors are consistent with the notion of Nash equilibrium and discusses some differences between differing beliefs and differing utilities.}\]
of an outsider with a ‘reference’ belief. This outsider can be interpreted as the board, the financial markets, or the researcher doing the analysis. The reference belief will be denoted $p$, which I assume $p \geq 1/2$ without loss of generality. I will say that a manager is visionary when her belief is stronger than the reference belief, i.e. $\mu_{M,A} > p$.

I further make two sets of assumptions.

**Assumption 1** Employees’ beliefs are independent draws from a distribution of beliefs $F$ on $[0,1]$, with continuous density $f$.

When indifferent about which firm to apply to, potential employees randomize between the firms with equal probability. When indifferent about what action to undertake, employees do as their manager prefers. When indifferent about implementation, managers do as their employee prefers.

**Assumption 2** The implementation cost $I$ is distributed uniformly on $[0,1]$. The cost of effort $c(e)$ is twice continuously differentiable with $c(0) = c'(0) = 0$; $c''(e) > 0$; $c(1) > \gamma_E \gamma_M$.

**Discussion of the model** The model implicitly assumes that all firms’ wages are identical and equal to zero and that the payoffs $\gamma_M$ and $\gamma_E$ are exogenously given. The reason for not considering a more explicit wage setting process is twofold. On the one hand, none of the many ways to model wage setting jumps out as the obvious choice in this context. On the other hand, the basic insights seem to be similar for all major alternatives, and this model with $w = 0$ has the most transparent results and least complicated analysis. While an earlier working paper showed how the $w = 0$, $\gamma_M$ and $\gamma_E$ can be interpreted as the outcomes of an up-front wage offer combined with ex-post bargaining, a serious study of wages requires an extended model with more firms and more uncertainty dimensions.\(^4\)

While $\gamma_E$ can thus be interpreted as the outcome of an ex-post renegotiation, there are other interpretations. A simple one is that $\gamma_E$ represents the agent’s private utility from a successful project. A more interesting version is that, with unobservable productivity, a successful project signals high productivity and will be compensated by higher wages in the market, which are then captured by $\gamma_E$.

The model also assumes that the employee selects the project type. I will discuss this assumption at the end of section 4 and consider the effect of having the manager select the project type.

Incontractibility is another important aspect of the model. In particular, the model implicitly assumes that the project type, the agent’s effort $e$, the fact that someone generates a project, and the future revenues from that project are economically too complex to

\(^4\)An earlier working paper also discussed some potential wage effects. Whether wages are higher or lower under a manager with strong beliefs depends on the distribution of bargaining power between employee and firm. The reason is that the manager is willing to pay higher wages, but attracts employees who are willing to work for lower wages. A second result is that, since many of the employees in a firm have stronger beliefs than the manager and the owners, it may make sense to pay them with stock options, in fact paying them with dreams. Moreover, the employees with the strongest beliefs also work hardest, so that there will be a non-causal correlation between getting stock options and expending effort.
contract on. This can be justified by the difficulty of describing a project that does not yet exist. Section 5.3 considers the effect of contractibility as a parameter. Appendix A further discusses how these contractibility assumptions affect the results.

One final remark facilitates the interpretation of the model. The distribution of beliefs can be interpreted as being generated by the following information process. All agents start with a common prior that both states are equally likely. All agents subsequently get a common signal that, for example, the true state is A, but have their own beliefs about the correctness of that signal. In particular, agent i believes that the signal is correct with probability $\mu_{i,A}$. Bayesian updating leads agent $i$ to believe that the probability of state A is $\mu_{i,A}$. The ‘reference’ belief $p$ is the board’s belief about the signal. Note that, in this interpretation, a ‘visionary’ manager is overconfident relative to the reference belief.

**A practical example** To fix ideas, think back to the time that the Internet was close to taking off and consider a software product manager who is preparing the next version of his product. His key issue is whether to improve traditional features or to add instead Internet capabilities. The success of his product may depend crucially on this choice. Complicating matters is the fact that the CEO has the final say on any new release. Consider now the case that the product manager believes the Internet is no more than a fad while his CEO may be a true believer.

In this case, contracting on direct output is problematic since it was difficult to define what Internet-ready means, what good implementation means, or what the relative importance of different features is, when the Internet was still evolving. Software development efforts are also difficult to measure objectively. Finally, his product’s success is obviously a key factor in the product manager’s future wage negotiations (or promotions), but it is difficult to contract on long in advance given the fundamental uncertainties in the industry.

**Definition of vision** Since this paper started out as an inquiry into the workings of managerial vision, the definition of vision is directly inspired by the original management literature on the topic. The term ‘vision’ was introduced in the management literature via research on leadership (Bennis, 1982; Bennis and Nanus, 1985) that built on the charismatic (House, 1977) and transformational (Burns, 1978) theories of leadership. Vision was defined as ‘a mental image of a possible and desirable future state of the organization’ (Bennis and Nanus, 1985) and having a vision was found to be a key discriminating characteristic of leaders. While most of the subsequent research-based literature stuck quite closely to this original meaning, the popular press and some more recent work have used the concept much

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5Note that differing beliefs about the correctness of the common signal is a special case of differing priors. Agents not only have (prior) beliefs about the state $x \in \{a, b\}$, but also about what game they are playing, how correct their information is, etc.

6In this context, it is interesting to note that just before Microsoft announced what later became known as the .NET initiative, Bill Gates handed the title of CEO to Steve Ballmer. The latter had a reputation of believing much more in the Internet than Gates himself.
more loosely.\textsuperscript{7}

This paper translates that ‘mental image of a desirable state of the organization’ into a more prosaic ‘very strong belief about the optimal future state of the organization’. While this captures the core elements of ‘vision’ as the term is used in the original literature, some characteristics that are mentioned in that literature are absent from the model. Most important is probably the claim that a vision should be attractive (Bennis and Nanus, 1985). Temporarily abstracting from such aspects is useful for two reasons. First of all, it results in a more transparent analysis. Second, the current results replicate very well the benefits that the managerial literature ascribes to vision (motivation, direction setting, consensus, coordination), which suggests that this belief aspect of vision is indeed one of its core elements.

As mentioned earlier, vision is thus defined in this paper as a strong belief of the manager about the right course of action for the firm. Given the setup of the model, this corresponds formally to the condition that $\mu_{M,A} \geq p$.

3 Preliminary Analysis: Decisions, Motivation, and Satisfaction

To understand the effect of managerial beliefs on employee effort and utility, I consider one specific employee and work by backwards induction. Given that the manager can observe the project type prior to implementation, she will implement a project $Y$ if and only if $\gamma M \mu_{M,Y} \geq I$, where $\mu_{M,Y}$ denotes the belief of the manager that the state is $Y$. A project of type $Y$ will thus be implemented with probability $\gamma E \gamma M \mu_{M,Y}$, which gives the employee an expected payoff $\gamma E \gamma M \mu_{M,Y}$ from developing the project. In choosing the project type and $e$, the employee thus solves:

$$\max_{e \in [0,1], Y \in \{A,B\}} e \gamma M \gamma E \mu_{M,Y} - c(e)$$

The next proposition says that whoever has the stronger belief about what should be done, will determine what the employee does. Denote the strength of belief by $\nu_i = \max(\mu_{i,A}, \mu_{i,B}) \in [1/2,1]$, i.e. $\nu_i$ is $i$’s belief in the state he considers most likely. An agent has a ‘stronger belief’ if $\nu_i$ is larger. Let $X$ denote the project type selected by the employee.

\textsuperscript{7}Kouzes and Posner (1987) define it as ‘an ideal and unique image of the future’; Kotter (1990) defined vision as ‘a description of something (...) in the future, often the distant future, in terms of the essence of what it should be.’ The Cambridge Dictionaries Online defines vision as ‘the ability to imagine how a country, society, industry, etc. will develop in the future and to plan in a suitable way’. A clear example of a widely expanded notion of vision is the work of Collins and Porras (1994) who essentially use the term ‘visionary’ as a more catchy synonym for ‘admired’. Such use of the term has given it a nearly magical meaning. People who are used to this more liberal use of the term, will consider the current definition very narrow.
**Proposition 1** If the manager has the stronger belief, i.e. $\nu_M \geq \nu_E$, then the employee undertakes the action that his manager prefers, i.e. $X = \arg\max_{Y \in \{A,B\}} \mu_{M,Y}$. Otherwise the employee follows his own opinion, i.e. $X = \arg\max_{Y \in \{A,B\}} \mu_{E,Y}$.

The intuition is as follows. If the manager and the employee agree on the optimal action, then $E$ chooses of course that action. If they have different opinions, the employee will have to ‘disappoint’ one of the two. Since the roles of their beliefs are symmetric in the employee’s utility function, it is optimal to ‘disappoint’ the one who holds the weaker belief (i.e. the belief closer to 1/2). While the simple form of this result depends on the specific assumptions of the model, the facts that the employee’s choice is influenced by the manager’s beliefs as well as by his own beliefs, and that stronger beliefs give the manager more influence, seem to be very robust.

Given the symmetry in the result, one might wonder what the difference is between the employee and the manager: why do managers have a vision while employees ‘only’ have beliefs? The difference is that, first, the manager influences the decision of the employee but not the other way around and, second, the manager also influences other employees.

A different way to look at proposition 1 is to say that the manager keeps a strong influence over the project type, even though the choice is formally delegated to the employee. Such indirect authority might, from the perspective of the manager, be more effective than direct authority, since the manager only has to get involved after the project has already been developed. For such decision processes, the earlier results imply the following.

**Corollary 1** (‘Visionary managers have more influence.’) The prior probability that the project choice is according to the manager’s belief increases in $\nu_M$, the manager’s belief strength.

While the manager’s opinion has an important influence on the decisions of the employee, it is also a key determinant for the employee’s effort and utility, i.e. his motivation and satisfaction. The following proposition says that a stronger belief of the manager will motivate the employee and increase his utility if the employee’s belief is such that he acts according to the manager’s beliefs. Stronger managerial beliefs, however, will demotivate and reduce the utility of an employee who goes against the manager’s opinion. To state this formally, let $N$ be an open neighborhood of $\mu_E$ and $\mu_M$ on which the chosen project type $X$ remains identical and let $0 < \mu, A < 1$ for both agents. Let $\hat{e}$ denote the employee’s optimal choice of effort while $\hat{u}$ is his maximized utility.

**Proposition 2** Employee effort $\hat{e}$ and utility $\hat{u}$ strictly increase in the conviction of the manager $\nu_M = \max(\mu_{M,A}, \mu_{M,B})$ (resp. in the employee’s own conviction $\nu_E$) on $N$ if the employee undertakes the action that the manager prefers, i.e. $X = \arg\max_{Y \in \{A,B\}} \mu_{M,Y}$ (resp. that he himself prefers).

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8 An extension of the model might also complement the theory of delegation (Prendergast, 1993; Aghion and Tirole, 1997; Baker et al., 1999; Zábojník, 2002). The main conjecture would be that, when effort is a complement to the likelihood of success, then the project type decision should be taken by the person with the more important non-contractible effort.
Analogously, employee effort $e$ and utility $u$ strictly decrease in the manager’s conviction (resp. his own conviction) on $N$ if he undertakes the opposite action of what his manager prefers, i.e. $X = \arg\min_{Y \in \{A, B\}} \mu_{M,Y}$ (resp. of what he himself prefers).

To see the intuition, suppose that the employee undertakes a project that is the right course of action according to his manager. As the manager is more convinced of that action, the probability that she will implement the project increases. This increases the expected payoff of the employee from developing the project, which indeed motivates him and gives him higher utility.

This result can be loosely interpreted as follows:

- Employees with no specific opinion on the correct action ($\mu_E$ close to 1/2) get more motivated by managers who know precisely what they want, no matter what they want. The same is true for employees whose utility depends only on implementation or approval, and not on the final success (since this case is formally equivalent to setting $\mu_{E,X} = 1$ for the likelihood of whichever action is chosen).

- Employees with a strong opinion about the correct path of action will be very motivated under managers who agree with them (and more so as the manager is more convinced of that opinion). But they will be very demotivated under managers with a different opinion.

These statements seem to fit casual empiricism.

4 The Sorting Effect

4.1 Basic analysis

The effects of beliefs on effort and utility cause sorting in the labor market. The basic argument runs as follows.

- Employees get higher utility working for firms that espouse a vision they agree with. Firms get higher profits from employees who agree with their vision, since the latter are more motivated. An efficient labor market should therefore match employees and firms with similar beliefs.

- Once sorting has taken place, the beliefs of the employees and the manager are more aligned. This will decrease or even eliminate the demotivating effect that vision had on some employees, so that vision becomes more effective.

9Note that effects similar to the ones described here can occur in other types of markets. In particular, investors (in financial markets) will be willing to pay more for equity in firms whose managers have beliefs that are similar to their own.
To study these effects formally, consider again the model of section 2 but let the employee, with belief $\mu_{E,A}$, have the choice between two firms, $F_1$ and $F_2$, with managers $M_1$ and $M_2$ who have beliefs $\mu_{M_1,A}$ and $\mu_{M_2,A}$, where I assume wlog. $\mu_{M_1,A} \geq \mu_{M_2,A}$. There is a unique subgame perfect equilibrium outcome, which gives sorting as indicated in figure 2.

**Proposition 3** Let $\bar{\mu} = \frac{1-\mu_{M_2,A}}{\mu_{M_1,A}+1-\mu_{M_2,A}}$. In any subgame perfect equilibrium, all employees with $\mu_{E,A} > \bar{\mu}$ go to $F_1$ and undertake $A$, while all employees with $\mu_{E,A} < \bar{\mu}$ go to $F_2$ and undertake $B$. $\bar{\mu}$ decreases in both $\mu_{M_1,A}$ and $\mu_{M_2,A}$.

![Figure 2: The choice of firm and action (indicated by the horizontal brackets) by the employees in function of the beliefs of both the two managers ($\mu_{M_1,A}$ and $\mu_{M_2,A}$) and the employees (whose beliefs are distributed over $[0,1]$ according to $F$) for two constellations of the managers’ beliefs (in top and bottom part of the figure).](image)

While the specifics of this proposition depend on the assumptions of the model, including the absence of explicit wage setting, the qualitative effects seem to be robust to such changes. This is also suggested by the fact that sorting is efficient.

To see intuitively what is happening in the proposition, consider first the upper graph of figure 2. All beliefs are represented along the horizontal line. There are two managers who have approximately opposite beliefs, $\mu_{M_1,A}$ and $\mu_{M_2,A}$, but with $M_1$ having slightly stronger beliefs. Consider now an employee with belief $\mu_E = 1/2$. Since project types $A$ and $B$ are equivalent in the eyes of this employee, he only cares about the probability of implementation. So he will join the firm with the manager with the strongest conviction, which is $M_1$. Given that his preference is strict and utilities are continuous, the cutoff $\bar{\mu}$ must be strictly to the left of $1/2$. Note now two things:

1. The employee with $\mu_E = 1/2$ is closer to $M_2$ in terms of beliefs, but goes to firm $F_1$, since $M_1$ ‘knows better what she wants’.

2. As $M_1$ gets more convinced, she becomes more attractive to work for. In particular, an employee that before was indifferent will now go to work for $M_1$. So $\bar{\mu}$ shift to the left as $\mu_{M_1}$ shifts to the right. The same is true for $M_2$. This gives the lower graph.
The following proposition says that the firm with the stronger vision attracts precisely those employees who take action according to its manager’s beliefs.

**Corollary 2** If manager $M_1$ has the stronger belief, i.e. $\nu_{M_1} > \nu_{M_2}$, then any employee who joins $F_1$ will choose the action preferred by its manager, i.e. $X = \arg\max_{Y \in \{A,B\}} \mu_{M_1, Y}$, while any employee who joins $F_2$ will choose the other action (which might or might not be preferred by $M_2$).

The intuition is that an agent who goes to $F_2$ and undertakes action $A$ would have been better off going to $F_1$ while still undertaking $A$, and vice versa.

The result also implies that firm 2 gets ‘pushed’ into undertaking $B$, even if its manager thinks $A$ is better. Firm 2 might thus be better off hiring a manager with the opposite vision of firm 1, or a manager whose vision is even stronger than $M_1$’s belief. This raises the issue how firms will compete on vision, which is outside the scope of the current paper.\(^\text{10}\)

Note that the model implicitly assumes that firms are not limited in size: they hire any employee who comes their way. This leads to the surprising result that the more visionary firm tends to be larger and have employees with more diverse beliefs. Typically, however, firms are not so flexible in terms of their size. Taking into account such limitations would make the results less extreme. The same is true if there were a larger state space.

The alignment of beliefs, both between a manager and her employees and among the employees, has broad implications. The next two subsections consider some basic implications for incentives and coordination. Further implications, for e.g. experimentation, learning, communication, conflict, or delegation, are outside the scope of the current paper.

### 4.2 The impact of sorting on incentives

In section 3, I concluded that stronger managerial beliefs could demotivate and reduce employees’ utility, that is, if they favored the other action so strongly that they act against the manager’s opinion. Sorting, however, will reduce this negative effect on incentives since potential employees who strongly disagree with the manager will go elsewhere. In particular, in the current setting the negative effect on incentives will be completely eliminated for the more visionary of the two firms since all its employees end up choosing according to the manager’s beliefs. Vision will thus always motivate.

**Corollary 3** If $M_1$ has the stronger belief ($\nu_{M_1} > \nu_{M_2}$) then the effort and utility of $F_1$’s employees increase in $\mu_{M_1, A}$ and thus in $\nu_{M_1}$.

### 4.3 The impact of sorting on coordination

The literature on vision has stressed its coordinating effects (Bennis and Nanus, 1985). While a full study of coordination goes beyond this paper, it is useful to consider how strong managerial beliefs might improve coordination in this setting. There are essentially 3 mechanisms through which vision aligns employees’ actions.

\(^{10}\) Analysis on a simple model suggests that such competition will often lead to even more extreme beliefs.
1. In choosing their actions, employees are influenced by their manager’s beliefs. Since all employees face the same managerial beliefs, their actions will become aligned.

2. Employees who take actions that fit the manager’s beliefs exert more effort than others, further strengthening the alignment.

3. The sorting effect directly aligns the beliefs of the employees. This also aligns their actions.

### 4.4 Visionary organizations

The analysis suggests the following characteristics of a ‘visionary organization’.

- Even when employees choose their projects without intervention from the top, they choose what management would want them to choose. This strengthens the case for delegation.
- The employees’ projects are aligned without any explicit coordination mechanism.
- Visionary firms also attract employees who do not agree with the vision, but who are attracted by its conviction.
- Vision motivates all employees, including those who think the other project is better.

As mentioned earlier, however, visionary organizations also have undesirable characteristics. In particular, managers with strong beliefs invest (ex-post) inefficiently, while their strong beliefs slow learning and adaptation.

### 4.5 Allocation of authority

This is also a good point to consider the allocation of authority in the model. The model assumes, in particular, that the employee chooses the project type. This is a realistic assumption in many settings. For new product designs or marketing campaigns, for example, general managers typically only get involved when the projects are in a fairly advanced stage.

There are, however, equally important settings where the project type is chosen up front by the manager and imposed on the employee.\(^\text{11}\) How does that change the results? Note first that a manager who believes in B might still choose A, since she cares how much effort her employee spends on the project. An employee who believes in A might also still prefer to go to a firm that makes him do B, if the probability of implementation is sufficiently higher. However, an employee who ends up working on A will always end up doing so in the firm with the manager who believes most in A. From all these arguments, it follows that all qualitative results will be preserved, though the cutoff points will be different, putting more weight on the manager’s beliefs.

\[^{11}\text{Even when the project type decision is taken on an intermediate level, the project type gets imposed on employees below that level.}\]
5 Profitability of vision

A key question remains: is vision good or bad? In other words, can we say anything about when a company gains from hiring a CEO with vision? In line with the discussion in section 2 on the outsider’s perspective, I will consider here the question ‘Given some reference \( p \), where we assume \( 1 > p > 1/2 \), is the optimal belief of the manager \( \mu_{M,A} > p \)?’. Remember that I abstract from the ‘dark side’ aspects of vision as mentioned in the introduction, i.e. I only consider whether a rational, well-informed, and correctly incentivized board should select a manager with beliefs that are stronger than its own.

The essential conclusions of this section are as follows. Absent sorting, vision is optimal for a restricted but important class of belief distributions. With sorting, vision is optimal under even more general conditions. Furthermore, the impact of vision decreases as the uncertainty goes to zero, but increases as incontractible actions become more important.

For the analysis it is useful to remember that, apart from the coordination effect, there are 3 forces in the model that determine the optimal CEO belief: the motivation/demotivation effect, the influence on the project choice, and the cost of wrong implementation decisions.

5.1 Profitability of vision absent sorting

Consider first the case without sorting. With employees’ beliefs drawn from a distribution \( F \), the firm’s reference expected profits can be written:

\[
E[\pi] = \int_{\mu_{M,B}}^{\mu_{M,B}} \hat{e}\gamma_{M,B}^2 \mu_{M,B} \left( (1 - p) - \frac{\mu_{M,B}}{2} \right) f(u)du + \int_{\mu_{M,B}}^{1} \hat{e}\gamma_{M,A}^2 \mu_{M,A} \left( p - \frac{\mu_{M,A}}{2} \right) f(u)du
\]

Since the balance of forces depends on the distribution of beliefs, I need further assumptions. Consider, in particular, the following important class of distributions that allows clear conclusions.

**Assumption 3** All agents think that \( A \) is the optimal project, i.e. \( \text{supp } F \subset (1/2, 1] \).

This assumption will, for example, be satisfied when all employees hold approximately the reference belief. It eliminates all employees who get demotivated or switch actions as the manager gets more convinced. The remaining trade-off is then between the motivation effect and the cost of wrong implementation.

**Proposition 4** Let A3 hold. If, in deviation from the general setup, the agent always generates a project, independent of his effort, then the unique optimal belief is the reference belief. If, as in the original setup, the probability of generating a project is \( e \), then vision is strictly optimal.

The intuition is that the motivation effect dominates as long as there is some role for effort, since the effect of wrong implementations is second order at \( \mu_{M,A} = p \). When the
motivation effect is completely absent, then the cost of wrong decisions makes the reference belief optimal.

For a practical example, let $\gamma_E = \gamma_M = 1/2$ and $c(e) = \frac{e^2}{8}$. In that case, the optimal belief is $\mu_{M,A} = \min(\frac{4p}{8}, 1)$ for any distribution of beliefs that satisfies A3.

5.2 Profitability of vision with sorting

When sorting occurs, an important cost of vision gets eliminated for the most visionary firm: no employee will get demotivated by the manager’s vision. Moreover, at small levels of overconfidence the cost of wrong implementations is second order since it concerns only projects that go marginally the other way. This suggests that ‘vision is always good in moderate amounts’. There is still one caveat, however: it is theoretically possible that all potential employees hold beliefs opposite to the reference belief $p$. A visionary firm ($\mu_M > p$) could then end up with nearly no employees and thus nearly no profits.

For the formal analysis, let the focal firm face one competitor whose manager holds the reference belief $p$. Consider the following alternate assumptions.

**Assumption 4** The support of $F$ is contained in $[(1 - p), 1]$.

**Assumption 5** The distribution of beliefs $F$ First Order Stochastically Dominates some symmetric distribution$^{12}$ and $1/2 < p < 1$.

This second assumption says that the distribution of beliefs weakly favors the side of the reference belief, in the sense that it can be generated from some symmetric distribution by moving some probability mass up. This holds, for example, when $F(x) \leq 1 - F(1 - x)$ or when $F$ is the Beta-distribution $F(x; a, b) = \int_0^x u^{a-1}(1-u)^{b-1}du / \int_0^1 u^{a-1}(1-u)^{b-1}du$ with $0 < b \leq a < \infty$.

The following results confirm that vision is optimal under either of the above assumptions.

**Proposition 5** Under either A4 or A5, vision is optimal (against a firm with $\mu_M = p$).

Figure 3 shows the optimal belief in function of $p$ when $\gamma_E = \gamma_M = 1/2$ and $c(e) = \frac{e^2}{8}$ and the employee beliefs are distributed $U[0, 1]$.

5.3 Comparative Statics

**Uncertainty** One would expect the impact of vision to decrease as the uncertainty about the true state goes to zero. The reason is that with less uncertainty, there is less room for a manager to be overconfident and thus for vision to make a difference. This would explain

---

$^{12}$A distribution $F$ first-order stochastically dominates a distribution $G$ when $F$ is generated from $G$ by adding to every outcome some non-negative random variable. An alternative definition is that $F \leq G$, i.e. some probability mass of $G$ is shifted upwards to obtain $F$.  

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15
why vision seems more important in high tech industries than, say, in the steel industry. And it is indeed what happens in the model.

With sorting, however, we have to be careful in stating the result. In this case, the gain from vision has two components. The first is the gain from inducing sorting with a minimum (limit) deviation from the reference belief, which I call the pure sorting effect. The second is the extra gain from holding a belief that is strictly greater than \( p \), which I call the gain beyond the pure sorting effect. The result applies only to the latter.

For the formal statement of the result, note that the measure for ‘uncertainty’ is \( p(1-p) \), the variance of the binomial distribution generated by the reference probability. The condition in the following proposition that \( p \rightarrow 1 \) thus captures the fact that the uncertainty decreases (given that I assumed \( p \geq 1/2 \)).

**Proposition 6**

- **Absent sorting**, the profit gain from vision, if any, converges to zero as \( p \rightarrow 1 \). Formally \( \lim_{\mu_{M,A} \rightarrow p} E[\pi] - E[\pi | \mu_{M,A} = p] \rightarrow 0 \) as \( p \rightarrow 1 \).

- **When vision is optimal under sorting**, the profit gain from vision beyond pure sorting converges to zero as \( p \rightarrow 1 \). Formally \( \lim_{\mu_{M,A} \rightarrow p} E[\pi] - \lim_{\mu_{M,A} \rightarrow p} E[\pi] \rightarrow 0 \) as \( p \rightarrow 1 \).

**Contractibility**  Another important parameter is the degree to which actions are contractible. To see its impact, consider the following extension of the model without sorting (section 5.1, including assumption 3). Let there be two parts to each job. The total payoff to the employee and to the firm is \( \alpha \) times the payoff from the first part of the job plus
\((1 - \alpha)\) times the payoff from the second part. The first part of the job is exactly as in the original model. The second part of the job differs in one important aspect from the original model: all decisions and revenue streams are contractible up front. Assume in particular that the manager makes the employee a take-it-or-leave-it offer about a contract on the second part of the job. If the employee rejects, the game continues as if only the first part of the job existed. Note that \(\alpha\) now measures the degree to which the actions are incontractible. The following proposition says that the optimal level of vision increases as the incontractible actions become more important, i.e. as \(\alpha\) increases.

**Proposition 7** For a given \(p\), the optimal \(\mu_M\) increases in \(\alpha\).

### 5.4 Why the Best and Worst Firms Are Always Visionary

Vision and strong beliefs also increase the variance of performance. If you act as if you knew the future and you turn out to be right, then your actions will be ex-post optimal, even if they were ex-ante suboptimal given the objective odds. This suggests that even when vision is not optimal, ex-post the best (but also worst) firms in the market will be those with visionary managers. This is especially important for empirical analyses of the effects of vision on firm performance, since it may induce a strong selection bias.

To confirm this argument formally, consider the following setting, which corresponds to the situation of the first part of proposition 4: each firm has one employee, all employees have reference beliefs \(\mu_E = p\), there is no sorting, and an employee always comes up with a project independent of his or her effort. Proposition 4 then says that it is optimal to hire a manager with the reference belief (‘no vision’). Consider now 4 managers with beliefs \(\mu_1 > 1/2\), \(\mu_2 = 1 - \mu_1\), \(\mu_3 = p\) with \(1/2 < p < \mu_1\), and \(\mu_4 = 1 - \mu_3\). I denote managers 1 and 2 as ‘visionary’ managers and the others as ‘regular’ managers since \(\nu_1 = \nu_2 > \nu_3 = \nu_4 = p\).

Let \(\pi_i\) denote the ex-post profits of firm \(i\). The following proposition says that ex post the best firm is a visionary firm, although ex ante its expected profit is lower than that of a regular firm.

**Proposition 8** \(\max(\pi_1, \pi_2) \geq \max(\pi_3, \pi_4)\) and \(E[\max(\pi_1, \pi_2) - \max(\pi_3, \pi_4)] > 0\) and increases in the uncertainty \(p(1-p)\). However, \(E[\pi_1] = E[\pi_2] < E[\pi_3] = E[\pi_4]\).

Alternatively, I could have showed that the proportion of visionary managers among the firms with the highest profits is higher than in the population at large, or that, with a sufficient number of firms and multiple projects per firm, almost surely the best firm is visionary.

This observation might explain why many famous ‘visionary’ managers were founders or co-founders of their firm (e.g. Steve Jobs, Sam Walton, Bill Gates, Larry Ellison, Scott McNealy): these people might actually have had too strong beliefs (from an ex ante perspective) but happened to be right. Such extreme believers will spend extreme effort on their ideas. The fact that this spurious effect will be stronger as there is more underlying uncertainty might also explain why 4 out of the 5 names above come from the software sector.
6 A Note on Testing the Theory

Testing the theory is the most important next step in terms of further research. The key challenge is to operationalize the concept of vision to an extent that it can be translated into measurable proxies. In particular, the theory is about beliefs and perceptions of beliefs, which we don’t observe.

One approach, though with many caveats, would be to start from the type of surveys that psychology uses in an attempt to measure beliefs and perceptions. A more indirect alternative would be to measure observable behaviors that are supposed to correlate with strong beliefs about the firm’s course of action, as, for example, in Malmendier and Tate (2003). At this point it is unclear whether either method can lead to a measure of vision with a sufficient degree of validity. It may therefore be necessary to further operationalize the concept to an extent that we can actually test the theory.

If satisfactory measures or proxies can be developed, then potential tests of the theory could be based on the prediction that vision is more important in industries with higher uncertainty and incontractibility or on the predicted relationship between, on the one hand, the congruence between the employee’s and management’s beliefs and, on the other hand, job selection, satisfaction, and motivation.

An experimental approach could be useful for the more modest objective of testing the sorting mechanism behind the model. A possible setup would be to put the subjects in the role of potential employees and face them with exactly the situation in the paper. To endow the subjects with a prior, they may get told the likelihood of success of the different projects and also be informed about the beliefs of the CEO’s, very much in the way that subjects get endowed with utility in auction experiments, such as Ariely, Ockenfels and Roth (2002).

Some related evidence While there are as yet no direct tests of the theory, there is some related evidence that is worth mentioning.

The first is evidence that a fit in terms of work values influences hiring, satisfaction, and turnover (Judge and Bretz, 1992; Meglino et al., 1989; Chatman, 1991; Chatman et al., 2001). While it is debatable how closely values and beliefs are related\textsuperscript{13}, this shows at least that job choice is not exclusively determined by wages, skills, and job description.

Another prediction of the theory is that a replacement of the CEO by someone with different beliefs should cause turnover immediately below her. If outsiders are more likely to have different beliefs than insiders, then succession by an outsider should also cause more turnover than by an insider. While this is consistent with the evidence of Hayes, Oyer and Schaefer (2002), there are other explanations for these observations, such as nepotism or career politics. On the other hand, however, the richer stories from the business press on mergers and acquisitions do support the idea that such turnover may be caused, at least in part, by differences in opinions about the right course of action for the firm.

\textsuperscript{13}A work value such as ‘customers first’ may be driven by an underlying belief that ‘customers first’ is more likely to lead to long-term success than, for example, ‘employees first’. There are, however, cases where the potential link is more tenuous.
7 Conclusion

This paper showed how vision, in the sense of a strong belief by the manager, causes sorting in the labor market that aligns the beliefs of employees with those of the manager, and indirectly with those of each other. This alignment has important consequences for incentives and coordination. The paper concluded that some degree of vision is profitable under relatively weak conditions, although it can also have its dark sides, and that the optimal beliefs are strongest under high uncertainty and low contractibility. Vision also increases the variance of performance, which may cause a selection bias.

The sorting mechanism in this paper makes the employees’ beliefs more homogenous. The relationship with corporate culture (Schein, 1985; Kotter and Heskett, 1992) is explored in more detail elsewhere (Van den Steen, 2004b). From that culture perspective, the mechanism in this paper suggests a role for leaders in the formation of culture.

Overall, the paper shows that a manager can have an important indirect influence on her firm’s behavior and performance. The most pressing need in terms of further research is to test the theory empirically. An important step in that direction would be to operationalize the notion of vision to a degree that we can measure it with available proxies. There are also potential theoretical extensions, especially the role of dynamic elements such as succession, learning, and communication. More broadly, the theoretical and empirical study of heterogenous beliefs may make a real contribution to our understanding of organizations and firms.
A Robustness of the Results and Potential Objections

A.1 Variations on the model

Consider first what happens when effort would become contractible. Assume in particular that the firm can offer an effort-based compensation \( b(e) \), which the employee can reject. If the employee rejects, the game just proceeds as before. The following informal argument suggests that all qualitative results are preserved in this case. Let \( \hat{e} \) and \( \check{e} \) denote the effort that the employee would choose with and without the extra compensation scheme. Any compensation scheme \( b(e) \) can be replicated by one that also induces \( \check{e} \) and that simply consists of a bonus \( \check{b} = b(\hat{e}) - b(\check{e}) \) if and only if the employee chooses \( e = \check{e} \). This bonus must be non-negative (since the employee can always reject \( b(e) \) and choose \( \hat{e} \) anyways). It must be that \( \check{e} \geq \hat{e} \) since the firm will never pay anything extra for a lower effort and that \( \check{b} = [\check{e} - \hat{e}]\mu_{E,Y}\gamma_{E,X} = [\check{e} - \hat{e}]c \) since this is the minimum to make the employee choose \( \check{e} \). The employee’s project choice and utility will thus be the same as in the original game, so that the satisfaction and sorting effects are preserved. The effort will be larger than before but still moves in the same way with the manager’s \( \nu \) thus be the same as in the original game, so that the satisfaction and sorting effects are preserved. The

If the project type were contractible, then the qualitative effects would again be preserved. While the manager’s beliefs will get more weight, the choice of project type will still be influenced by both beliefs (since the manager wants to motivate the employee to spend effort). The employee’s utility and effort also still depend on his own and his manager’s beliefs, so that there will be sorting.

Consider now some more structural changes to the model. Consider first the role of employee effort \( e \). In particular, in the model employee utility was strictly increasing and supermodular in \( e, \mu_{E,Y} \) and \( \mu_{E,Y} \). While this appears to be the more natural case, these properties do not necessarily always hold in modified games\(^{14}\). The property that the employee’s utility increases in the manager’s belief in his project, seems to hold in most situations. In that case, vision still causes sorting and an increase in utility. The complementarity between \( e \) and \( \mu_{E,Y} \) however, is more fragile. In some situations, the motivation effect may get lost or even reversed. If so, the optimality of vision depends on the exact strength and interaction of the different effects.

A.2 Potential objections

A potential issue is the fact that many decisions are taken without the CEO’s involvement. Sorting, however, happens even if a CEO intervenes only sporadically. Moreover, CEO’s do get involved in the key decisions, which are typically the ones employees care most about. Finally, there is also a cascading effect: the CEO’s beliefs cause a sorting on the level just below him, which then leads to similar effects on the next level.

But wouldn’t it be easier or more effective to relax the budget procedures (to accept more projects) instead of hiring a visionary manager? Budgeting tweaks, however, will not cause any sorting unless the project type is contractible. Moreover, the budget procedures rely on estimates of future revenues and costs, which are by definition subjective and thus dependent on managerial beliefs.\(^{15}\) Finally, there is the problem of commitment. Budget approval procedures are usually under the authority of the CEO, who can overrule them at any time. So, in the end, the manager’s beliefs will matter anyways.

---

\(^{14}\)Consider, for example, the following modification. Let the cost of implementation be distributed according to some general distribution function \( G \). Let \( e \) now be the probability that the employee’s project will be a success conditional on being of the right type (i.e. conditional on fitting the state), instead of the probability that the employee comes up with a proposal. In this case, the employee’s overall utility function becomes \( \gamma_{E,Y}eG(\gamma_{M,\mu_{E,Y}}e) \). Complementarity between \( \mu_{E,Y} \) and \( e \) now depends on the behavior of \( g' \).

\(^{15}\)Motorola thought that there was a huge market for globally functioning mobile phones, even if the phone weighed a few pounds. More restrictive budget procedures probably wouldn’t have prevented them from sinking $6 billion dollars in the Iridium project.
Why can’t CEO’s simply ‘proclaim’ the vision without actually holding extreme beliefs? This is again an issue of commitment: if lower level managers don’t expect the CEO to follow up on her claims, then they will also not change their actions. The only real commitment, apart from contracting, comes from the manager’s beliefs.
B Proofs of the propositions

Proof of Proposition 1: Since the employee solves \( \max_{e \in [0,1], Y \in \{A,B\}} \gamma_{E,Y} \mu_{E,Y} \mu_{E,Y} - c(e) \) he chooses the project \( X \) with the highest \( \mu_{E,Y} \mu_{E,Y} \), and then solves \( \max_{e \in [0,1]} \gamma_{E,Y} \mu_{E,X} \mu_{E,X} - c(e) \) he chooses the optimal which is strictly positive. This implies the first part of the statement. The second part follows from applying \( \max \).

Proof of Proposition 2: \( F \) employees choose \( A \).

Proof of Corollary 1: \( \mu_{M,A} > 1/2 \), so that \( \nu_M = \mu_{M,A} \). The probability that the decision follows the manager’s belief is \( F \) which increases in \( \mu_{M,A} \) and thus in \( \nu_M \).

Proof of Proposition 3: \( \mu_{E,A} > \mu_{M,A} \), so that \( \nu_M = \mu_{M,A} \). If now \( \nu_M = \mu_{M,A} \), then \( \mu_{M,X} = \mu_{M,B} = 1 - \mu_{M,A} = 1 - \nu_M \), so that \( \frac{d\mu_{M}}{d\nu_M} = \frac{d\nu_M}{d\mu_M} = -\frac{d\mu_{M}}{d\nu_M} \) < 0. The other arguments are analogous.

Proof of Corollary 2: This follows directly from the proof of proposition 3.

Proof of Corollary 3: By the earlier results, all employees of \( F_1 \) choose \( A \). The corollary then follows from monotone comparative statics and an envelope theorem on the employee’s problem.

Lemma 1: Absent sorting, the optimal \( \mu_{M,A} \) increases in \( p \).

Proof: It is sufficient to show that \( E[\tilde{\pi}_O] \) is supermodular in \( p \) and \( \mu_{M,A} \). The profit equation is:

\[
E[\tilde{\pi}_O] = \int_{p=0}^{\mu_{M,B}} \tilde{e}_{\mu_{M,A}} \left(1 - p\right) \mu_{M,B} - \frac{\mu_{M,A}^2}{2} \right) f(u)du + \int_{\mu_{M,B}}^{1} \tilde{e}_{\mu_{M,A}} \left(\mu_{M,B} - \frac{\mu_{M,A}^2}{2}\right) f(u)du
\]

where I suppressed notation that indicates that \( \tilde{e} \) depends on both agents’ beliefs and on the action taken. The cross partial of this function in \( (p, \mu_{M,A}) \) is positive.

Proof of Proposition 4: \( \mu_{M,A} > 1/2 \). By lemma 1 above, it is sufficient to show this for \( p = 1/2 \). By contradiction, assume that \( \mu_{M,A} < 1/2 \) while \( p = 1/2 \), then firm profits are:

\[
E[\tilde{\pi}_O] = \int_{1/2}^{\mu_{M,B}} \tilde{e}_{\mu_{M,A}} \left(\mu_{M,B} - \frac{\mu_{M,A}^2}{2}\right) f(u)du + \int_{\mu_{M,B}}^{1} \tilde{e}_{\mu_{M,A}} \left(\mu_{M,B} - \frac{\mu_{M,A}^2}{2}\right) f(u)du
\]

Consider now what happens if we select instead a manager with belief \( \mu_{M,A} = 1 - \mu_{M,A} > 1/2 \).
Employees who before chose A will still choose A, but their effort strictly increases. This implies that the second term strictly increases.

Employees who before chose B will now choose A. By the relation between \( \mu_{M,A} \) and \( \hat{\mu}_{M,A} \), the \( \mu_{E,X} \) (the manager’s belief in the action chosen by the employee) remains the same. \( \mu_{E,X} \) on the contrary increases (since by A3 all employees believe more in A than in B), so that again employee effort increases. This implies that the first term increases.

This implies that overall the firm profits increase, so that \( \mu_{M,A} < 1/2 \) is not optimal.

Consider now the case that the employee always comes up with a project, independent of his effort. The employee sets \( \hat{e} = 0 \) and undertakes the action that maximizes \( \mu_{E,Y} \mu_{M,Y} \). Since \( \mu_{M,A} \geq 1/2 \), profit equals \( E[\hat{\pi}_O] = \int_{1/2}^{1} \gamma_2 \mu_{M,A} \left( p - \frac{\mu_{M,A}}{2} \right) f(u)du \) which is maximized at \( \hat{\mu}_{M,A} = p \). This proves the first part of the proposition. For the second part, the firm profit when \( \mu_{M,A} \geq 1/2 \) is \( E[\hat{\pi}_O] = \max_{\mu_{M,A} \geq 1/2} \int_{1/2}^{1} \gamma_2 \mu_{M,A} \left( p - \frac{\mu_{M,A}}{2} \right) f(u)du \) where the maximum is well defined since the profit function is continuous in \( \mu_{M,A} \) on \([1/2,1]\]. The derivative of the integrand (for \( \mu_{M,A} \)) is strictly positive for \( 1/2 \leq \mu_{M,A} \leq p \) and continuous in \( \mu_{M,A} \). It thus follows that the optimal \( \mu_{M,A} \) is strictly larger than \( p \) and thus that vision is optimal.

For the profitability with sorting, remember that I assumed that \( 1 > p > 1/2 \) and that the focal firm \( F \) faces one competitor with belief \( \mu = p \). Let us first introduce some notation. Let \( \hat{\pi}_L = \max_{\mu_{F,A} \geq p} E[\hat{\pi}] \) when \( F \) attracts all employees with \( \mu_{E,A} \geq \hat{\mu} \), and let \( \hat{\mu}_{F,L} \) be the corresponding maximizer. Let analogously \( \hat{\pi}_L = \max_{\mu_{F,A} \leq p} E[\hat{\pi}] \) when \( F \) attracts all employees with \( \mu_{E,A} \leq \hat{\mu} \), and let \( \hat{\mu}_{F,L} \) be the maximizer. Note that this implies that \( 0 \leq \hat{\mu}_{F,L} \leq p \leq \hat{\mu}_{F,H} \leq 1 \).

Let \( \hat{\pi}_L \) be the profit of \( F \) when \( \mu_{F,A} = \hat{\mu} \) but \( F \) attracts all employees with \( \mu_{E,A} < (1 - p) \); \( \hat{\pi}_H \) be the profit of \( F \) when \( \mu_{F,A} = \hat{\mu} \) and \( F \) attracts all employees with \( \mu_{E,A} \geq (1 - p) \); \( \hat{\pi}_H \) be the profit of \( F \) when \( \mu_{F,A} = p \) and employees are allocated randomly between the two firms with equal probability. Note that it is always true that \( \hat{\pi}_H \geq \hat{\pi}_H \) and \( \hat{\pi}_L \geq \hat{\pi}_L \). Finally, let \( F^-(x) = \lim_{u \downarrow x} F(u) \) and \( F^+(x) = \lim_{u \uparrow x} F(u) \).

**Lemma 2** If \( F^-(1 - p) < 1 \) then \( \hat{\mu}_{F,H} > p \). If \( F^+(1 - p) > 0 \) then \( \hat{\mu}_{F,L} < p \). Finally, if \( F^-(1 - p) < 1 \) or \( F^+(1 - p) > 0 \) then either \( \hat{\pi}_H > \hat{\pi}_H \) or \( \hat{\pi}_H > \hat{\pi}_H \) or both. If both conditions are satisfied (which is the case when \( F \) has full support), then the optimal belief is strictly different from the reference belief.

**Proof:** Consider the first part of the lemma, so assume \( 1 - F^-(1 - p) > 0 \). Conditional on \( \mu_{F,A} \geq p \) and \( F \) attracting all employees with \( \mu_{E,A} \geq \hat{\mu} \), its optimal profits are

\[
\hat{\pi}_H = \max_{\mu_{F,A} \geq p} \int_{1}^{\hat{\mu}} \gamma_2 \left( p \mu_{F,A} - \frac{\mu_{F,A}}{2} \right) f(u)du
\]

with \( \hat{\mu} = \frac{1 - p}{\mu_{F,A} + 1 - p} \). This profit function is (right)continuously differentiable in \( \mu_{F,A} \) on \([p,1]\). Its right derivative in \( \mu_{F,A} \) at \( \mu_{F,A} = p \) is:

\[
\left. \frac{d\hat{\pi}_H}{d\mu_{F,A}} \right|_{\mu_{F,A} = p} = \int_{1-p}^{1} \gamma_2 \left( p^2 \frac{d\hat{\pi}_H}{d\mu_{F,A}} f(u)du - \gamma_1 \right) f(1-p) \]

The second term is non-negative since \( \frac{d\hat{\pi}_H}{d\mu_{F,A}} \leq 0 \). The first term is strictly positive since \( F(1 - p)^- < 1 \) and \( \frac{d\hat{\pi}_H}{d\mu_{F,A}} > 0 \). This implies that the optimal \( \hat{\mu}_{F,H} > p \). Note that this also implies that \( \hat{\pi}_H > \hat{\pi}_H \).

The argument for the second part is analogous and implies \( \hat{\pi}_L > \hat{\pi}_L \).

I now show that if \( F^-(1 - p) < 1 \) or \( F^+(1 - p) > 0 \) then either \( \hat{\pi}_L > \hat{\pi}_L \) or \( \hat{\pi}_H > \hat{\pi}_H \) or both. Just checking definitions of \( \hat{\pi}_L, \hat{\pi}_H, \) and \( \hat{\pi}_L \) shows that \( \hat{\pi}_L + \hat{\pi}_H = 2 \hat{\pi}_M \). But, it is always true that \( \hat{\pi}_H \geq \hat{\pi}_H \) and \( \hat{\pi}_L \geq \hat{\pi}_L \).
with one of these strict when \( F^- (1 - p) < 1 \) or \( F^+ (1 - p) > 0 \). This implies that under that condition \( \hat{\pi}_L + \hat{\pi}_H > \bar{\pi}_L + \bar{\pi}_H = 2\bar{\pi}_M \) which implies that \( \max (\hat{\pi}_L, \hat{\pi}_H) > \bar{\pi}_M \).

The very last part follows from the fact that when \( F^- (1 - p) < 1 \) and \( F^+ (1 - p) > 0 \) then \( \hat{\mu}_H > p \) and \( \hat{\mu}_{FL} < p \).

**Proof of Proposition 5:** For A5, this follows immediately from the lemmas that follow. For A4, it is immediate that the optimal belief must be \( \mu \geq p \) since a firm with \( \mu < p \) has no employees. Next, there exist some \( \mu > p \) that gives the focal firm higher profits than \( \mu = p \) (since with \( \mu > p \) all the employees prefer the focal firm, while they randomize between the two when \( \mu = p \)). Finally, the right-derivative (in the manager's belief) of firm profit at \( \mu = p \) is strictly positive, so that the optimal belief subject to \( \mu \in (p, 1] \) is well-defined.

**Lemma 3** *Vision is optimal (against a firm with reference beliefs) for any symmetric distribution of beliefs.*

**Proof:** Fix a symmetric distribution of beliefs \( F \). Note that it is always true that \( F^- (1 - p) < 1 \), so that \( \hat{\mu}_{FL} > p \). Consider first the case that \( p = 1 - p = 1/2 \). By symmetry \( \bar{\pi}_L = \bar{\pi}_H \) so that vision (\( \hat{\mu}_p > p \)) is (weakly) optimal. As \( p \) increases, \( \bar{\pi}_H \) strictly increases since \( \frac{d \bar{\pi}_H}{dp} = \frac{\partial \mu}{\partial p} \int_0^1 e^{\gamma F_{FL, H}} f(u) du > 0 \), while \( \bar{\pi}_L \) (weakly) decreases since \( \frac{d \bar{\pi}_L}{dp} = \frac{\partial \mu}{\partial p} = -\int_0^1 e^{\gamma F_{FL, H}} f(u) du \leq 0 \). This implies that for all \( p > 1/2 \), \( \bar{\pi}_H > \bar{\pi}_L \).

**Lemma 4** Let \( G \) and \( H \) be distribution functions on \([a, b]\), with \( H \) FOSD \( G \). Let further \( k (\theta, x) = E_{\theta \sim H + (1 - \theta) G} [f_3(x, u) | f_1(x) \leq u \leq f_2(x)] \) with \( \theta \in [0, 1] \), \( a \leq f_1 \leq f_2 \leq b \) and \( f_3 \) \( u \)-measurable. Let finally \( K (\theta) = \max_{x \in X} k (\theta, x) \) be well-defined for \( \theta \in [0, 1] \). If \( f_3(x, u) \) increases in \( u \) (for fixed \( x \)), then \( K (1) \geq K (0) \).

**Proof:** Let \( f_3(x, u) \) be increasing in \( u \). Since \( H \) FOSD \( G \), the basic theorem on FOSD says that for any fixed \( x \in X \), \( k (1, x) = E_{\theta \sim H} [f_3(x, u) | f_1(x) \leq u \leq f_2(x)] \) \( \geq E_{\theta \sim G} [f_3(x, u) | f_1(x) \leq u \leq f_2(x)] = k (0, x) \). Let \( \hat{\pi}_H \in \operatorname{argmax}_{x \in X} k (1, x) \) and \( \hat{\pi}_G \in \operatorname{argmax}_{x \in X} k (0, x) \) which exist by assumption. It then follows that \( K (1) = k (1, \hat{\pi}_H) \geq k (1, \hat{\pi}_G) = K (0) \) which proves the lemma.

**Lemma 5** If vision is optimal for some belief-distribution \( G \), then it is optimal for any belief-distribution \( H \) that \( G \) FOSD \( H \).

**Proof:** The fact that vision is optimal for some belief-distribution \( G \) implies that \( \hat{\pi}_{H, G} \geq \hat{\pi}_{L, G} \) where

\[
\hat{\pi}_{H, G} = \max_{\mu_{FL} \geq p} \int_0^1 \hat{\gamma}_{FL}^2 \left( p \mu_{FL} - \frac{\mu_{FL}^2}{2} \right) g(u) du \quad \text{with} \quad \hat{\mu}_L = \frac{1 - \mu_{FL}}{\mu_{FL} + 1} \\
\hat{\pi}_{L, G} = \max_{\mu_{FL} \geq 1 - p} \int_0^1 \hat{\gamma}_{FL}^2 \left( (1 - p) \mu_{FL} - \frac{\mu_{FL}^2}{2} \right) g(u) du \quad \text{with} \quad \hat{\mu}_L = \frac{1 - \mu_{FL}}{\mu_{FL} + 1}.
\]

Define now \( \tau_H (\mu_{FL}, p, \hat{\mu}_L, u) = \hat{\gamma}_{FL}^2 \left( p \mu_{FL} - \frac{\mu_{FL}^2}{2} \right) \) if \( u \geq \hat{\mu}_L \) and zero otherwise. Define analogously \( \tau_L (\mu_{FL}, p, \hat{\mu}_L, u) = \hat{\gamma}_{FL}^2 \left( (1 - p) \mu_{FL} - \frac{\mu_{FL}^2}{2} \right) \) if \( u \leq \hat{\mu}_L \) and zero otherwise. It then follows that \( \hat{\pi}_{H, G} = \max_{\mu_{FL}} \int_0^1 \tau_H (\mu_{FL}, p, \hat{\mu}_L, u) g(u) du \) and \( \hat{\pi}_{L, G} = \max_{\mu_{FL}} \int_0^1 \tau_L (\mu_{FL}, p, \hat{\mu}_L, u) g(u) du \). By Lemma 4 it suffices to show that \( \tau_H \) increases and \( \tau_L \) decreases in \( u \), to conclude that \( \hat{\pi}_{H, G} \geq \hat{\pi}_{H, G} \geq \hat{\pi}_{L, G} \) which would imply the proposition. The rest of this proof shows that that is indeed the case.
Note first that the optimal $\mu_{FH}$ and $\mu_{FL,B}$ must be such that
\[
(p\mu_{FH} - \frac{\mu_{FH}^2}{2}) > 0 \quad \text{and} \quad ((1-p)\mu_{FL,B} - \frac{\mu_{FL,B}^2}{2}) > 0
\]
since otherwise profits are non-positive while, in each case, it is always possible to set $\mu_F = p$, which gives strictly positive profits. But then the inequalities follow immediately: For $\tau_H$ (using the fact that $\hat{\mu}_H$ is no function of $u$): the derivative is zero for $u < \hat{\mu}_H$, the function makes a jump upwards at $\hat{\mu}_H$, and the derivative for $u > \hat{\mu}_H$ is $\gamma_p \left( p\mu_{FH} - \frac{\mu_{FH}^2}{2} \right) \frac{d\hat{\mu}}{du}$ which is positive (since $\frac{d\hat{\mu}}{du}$ is positive for employees who undertake $A$). An analogous argument for $\tau_L$ shows that it is decreasing. ■

Proof of Proposition 6: For the first part of the proposition, note that if $\hat{\mu}_M = \arg\max_{\mu_M,A \geq p} (E[\pi])$, then $p \leq \hat{\mu}_M \leq 1$. Clearly, as $p \uparrow 1$, $\hat{\mu}_M \uparrow 1$. This combined with the continuity of the expected profit $E[\hat{\pi}_M]$, implies the proposition.

For the second part of the proposition, note that vision is optimal so that $p < \hat{\mu}_M \leq 1$. Clearly, as $p \uparrow 1$, $\hat{\mu}_M \rightarrow 1$. But this, combined with continuity of the profit function $E[\hat{\pi}_M]$, implies the proposition. ■

Proof of Proposition 7: Let $\hat{\mu}_\alpha$ denote the optimal belief. The value to the firm of the second part of the job is
\[
\hat{\epsilon}_\gamma (p - \hat{\mu}_\alpha) V + \left[ \hat{\epsilon} \frac{\gamma^2}{2} - c(\hat{\epsilon}) \right]
\]
where $V = \gamma_M \mu_M + \gamma_H \mu_H$ and $\hat{\epsilon} = \arg\max_{\epsilon} \epsilon \frac{\gamma^2}{2} - c(\epsilon)$. This can be shown to increase in $\mu_M$ for $\mu_M < p$ and decrease in $\mu_M$ for $\mu_M > p$. Combined with the result of section 5.1, it follows that $p \leq \hat{\mu}_\alpha$. To see that the result then follows, fix some $\tilde{\alpha} < \hat{\alpha}$. The fact that $\bar{\mu}_\alpha$ is optimal for $\hat{\alpha}$ means that all $\mu_M > \bar{\mu}_\alpha$ are inferior for $\hat{\alpha}$: the (potential) gain on the first task does not compensate the (sure) loss on the second task. But for the smaller $\tilde{\alpha}$, the (potential) gain on the first task gets less important and the (sure) loss on the second task gets more important, so it follows that for the smaller $\tilde{\alpha}$ these $\mu_M > \bar{\mu}_\alpha$ are even more inferior, so that $\tilde{\mu}_\alpha \leq \bar{\mu}_\alpha$. This implies the result. ■

Proof of Proposition 8: Conditional on $A$ being correct, the ex-post difference in profit $\max(\pi_1, \pi_2) - \max(\pi_3, \pi_4)$ equals
\[
(\gamma_M - I) \left( 1_{(\gamma_M \leq t)} - 1_{(\gamma_M > t)} \right)
\]
with 1 denoting the index function. This is everywhere non-negative, while its expected value is strictly positive and increases as $p$ decreases. The result conditional on $B$ being the right project is analogous. The combination implies the first part of the proposition. The second part follows from proposition 4. ■
References


