Summary of WKB Connection Formulas

Here is a summary of the connection formulas for the two cases corresponding to turning points on the right and left of an allowed region:

I. Forbidden region to the right (turning point at \( x_0 = a \)).

- If the wavefunction is known to be exponentially falling in the forbidden region, then its phase and amplitude are known in the allowed region:

\[
\frac{1}{\sqrt{\kappa(x)}} \exp \left[ -\frac{1}{\hbar} \int_a^x \kappa(x')dx' \right] \Rightarrow \frac{2}{\sqrt{p(x)}} \cos \left[ \frac{1}{\hbar} \int_x^a p(x')dx' - \frac{\pi}{4} \right]
\]

(1)

- A wavefunction 90° out of phase in the allowed region continues into a growing exponential in the forbidden region as follows:

\[
\frac{1}{\sqrt{p(x)}} \cos \left[ \frac{1}{\hbar} \int_x^a p(x')dx' + \frac{\pi}{4} \right] \Rightarrow \frac{1}{\sqrt{\kappa(x)}} \exp \left[ \frac{1}{\hbar} \int_a^x \kappa(x')dx' \right]
\]

(2)

II. Forbidden region to the left (turning point at \( x_0 = b \)).

- If the wavefunction is known to be exponentially falling in the forbidden region, then it’s phase and amplitude are known in the allowed region:

\[
\frac{1}{\sqrt{\kappa(x)}} \exp \left[ -\frac{1}{\hbar} \int_x^b \kappa(x')dx' \right] \Rightarrow \frac{2}{\sqrt{p(x)}} \cos \left[ \frac{1}{\hbar} \int_b^x p(x')dx' - \frac{\pi}{4} \right]
\]

(3)

- A wavefunction 90° out of phase in the allowed region continues into a growing exponential in the forbidden region as follows:

\[
\frac{1}{\sqrt{p(x)}} \cos \left[ \frac{1}{\hbar} \int_b^x p(x')dx' + \frac{\pi}{4} \right] \Rightarrow \frac{1}{\sqrt{\kappa(x)}} \exp \left[ \frac{1}{\hbar} \int_x^b \kappa(x')dx' \right]
\]

(4)