A Brief Discussion of Dipole Fields

For the purposes of 8.02, we don’t need to use fancy vector calculus to find the field of an ideal dipole on the axis of the dipole or in the plane perpendicular to the dipole.

Let’s start with an electric dipole. Take a look at Example 22-14 on Page 627, where the field on the dipole axis is given by

\[ E = \frac{p}{2\pi \epsilon_0 z^3} \]

for \( z \gg l \), where \( l \) is the distance between the charges which form the dipole and I’ve used \( z \) instead of \( y \) for the coordinate along the axis.

For a point on the \( x \)-axis, the calculation is simpler. Example 22-9 on Page 620 finds the field for a specific value of the distance. Let’s reproduce this for an arbitrary distance. From the figure below, the electric field a distance \( x \) from the dipole is seen to be

\[ \overrightarrow{E} = -\hat{k} \frac{2q}{4\pi \epsilon_0} \frac{1}{x^2 + l^2} \frac{l}{\sqrt{x^2 + l^2}} = -\hat{k} \frac{p}{4\pi \epsilon_0} \frac{1}{(x^2 + l^2)^{3/2}} = -\frac{\vec{p}}{4\pi \epsilon_0} \frac{1}{(x^2 + l^2)^{3/2}}. \]

If \( x \gg l \), \( \overrightarrow{E} = (-\frac{\vec{p}}{4\pi \epsilon_0}) (1/x^3) \).

Now, consider a magnetic dipole, modeled by a current loop of radius \( a \) carrying a current \( I \). For the field on the axis, refer to Equation (29-16) on Page 815, but
using \( z \) for the axis coordinate instead of \( x \) and taking \( N = 1 \). For \( z \gg a \), this component of the field becomes

\[
B = \frac{\mu_0 I a^2}{2 z^3} = 2 \left( \frac{\mu_0}{4\pi} \right) \frac{I \pi a^2}{z^3} = 2 \left( \frac{\mu_0}{4\pi} \right) \frac{\mu}{z^3},
\]

where \( \mu = I \pi a^2 \), the product of the current and the loop area, is the magnetic moment (and it certainly was not my idea to use the symbol “\( \mu \)” for this physical quantity). So, we see that we reproduce the previous result for \( E \) on the electric dipole axis, with \( p \rightarrow \mu, 1/4\pi\varepsilon_0 \rightarrow \mu_0/4\pi \).

At this point, we can and will use an analogy to the electric dipole to state that the magnetic field in the plane of the dipole perpendicular to its axis is

\[
\vec{B} = -2 \left( \frac{\mu_0}{4\pi} \right) \frac{\vec{I}}{x^3}
\]

for \( x \gg a \).

The above may be derived slightly more rigorously, but that derivation will be given separately, and is completely optional.