Using MAPLE to Solve Direct-Curent Circuit Problems

These notes contain instructions and hints on how to use MAPLE to do the sort of linear algebra that is often encountered in dealing with DC circuits. As mentioned, YHI finds this a real convenience, and once you get used to MAPLE or some similar program, you might find yourself saving a lot of time, sparing yourself lots of algebra that can be done by computer. You can also amaze your friends and confound your enemies.

The problem used as an example is Challenge Problem 27-75 in the Ninth Edition, 27-55 in the Eighth. After going through the procedure outlined below, you might see that what is labeled as a “challenge” is subjective at best. A large part of the problem is showing how the problem may be divided into parts in a way that allows solution of similar problems without a great deal of recalculation. MAPLE can be used in such a way to show this division explicitly, and to do pretty much all of the work. I’ll show how to do the problem three ways, using varying degrees of sophistication on MAPLE.

The Circuit

As indicated in the problem, we wish to find the currents in the three branches of the circuit. The “obvious” choice of loops for the loop law, the left and right branches, leads to the three linear algebraic equations

\[ I_1 R_1 + I_2 R_2 = E_1 + E_2 \]
\[ I_2 R_2 + I_3 R_3 = E_2 + E_3 \]
\[ I_1 + I_3 - I_2 = 0 \]

where the resistors are identified symbolically and \( E_1, E_2 \) and \( E_3 \) are the voltages in the left, middle and right branches, respectively.

Introduction to MAPLE

These instructions are for an X-term. MAPLE will run on other terminals, but the graphics may not be as good. In fact, if you try to run the things I describe here on a terminal that can’t handle the graphics, MAPLE will call your terminal “stupid”. Really. (The things that you should have on your screen are in typewriter font. Things such as the athena prompt (%) and the MAPLE prompt (>) are included here, but of course need not be typed.)
The commands given here should run on any release of MAPLE; download the appropriate worksheet or view the ASCII file from the links on the web page.

To begin, start MAPLE from the Dash/Console menu bar under Numerical/Math, Analysis and Plotting. Or, at the Athena prompt,

```
> a1:=140*i1+210*i2=55+92;
> a2:=35*i3+210*i2=55+57;
> a3:=i1+i3=i2;
```

Wait a bit; MAPLE is a large program, and it takes a while to load. If you want to load an earlier version, which loads more quickly and has spiffier graphics (the problem in question here is spiffless), do

```
> a1:=140*i1+210*i2=55+92;
> a2:=35*i3+210*i2=55+57;
> a3:=i1+i3=i2;
```

When the MAPLE prompt, >, appears, MAPLE is ready to run. When the MAPLE window appears, notice the “Help” menu at the upper right. If you are new to MAPLE, so that MAPLE is new to you, either type

```
> ?introduction
```

and return at the MAPLE prompt or select the proper category from the Help menu.

For our present purposes, we won’t need any new syntax until and unless you want to try the Linear Algebra package, as indicated below. If you prefer, you can cut and paste the commands directly into your MAPLE session from the ASCII file

```
/mit/8.01-esg/8.02/F03/notes/dctempa
```

or download from the web. This file includes the MAPLE prompt, “>”, which you should not copy into your worksheet.

**Quick ’n’ Easy**

Suppose you just want to solve the problem at hand. That’s fine. Use the equations in ♣ above, but with numbers, as in

\[
\begin{align*}
I_1 \ 140.0 \ \Omega + I_2 \ 210.0 \ \Omega &= 92.0 \ V + 55.0 \ V \\
I_2 \ 210.0 \ \Omega + I_3 \ 35.0 \ \Omega &= 55.0 \ V + 57.0 \ V \\
I_1 + I_3 - I_2 &= 0.
\end{align*}
\]

MAPLE doesn’t care about units, even though we do. These equations are entered into your MAPLE session as

```
> a1:=140*i1+210*i2=55+92;
> a2:=35*i3+210*i2=55+57;
> a3:=i1+i3=i2;
```
and notice a few things. First, the use of \(a_1\), \(a_2\) and \(a_3\) to identify equations is a convenience only, and not really part of the program or the physics. I’m using lower case letter for the currents just so that I can use upper case later on; MAPLE is case sensitive. If you enter the equations yourself, you’ve got to get every single colon and semicolon correct; MAPLE is very unforgiving of typos. Note that the last equation is algebraically identical to the corresponding equation in ♣ or ♦, but you knew that.

Now comes the fun; the command to solve is just that,

\[
>\text{solve}\{a_1,a_2,a_3\},\{i_1,i_2,i_3\};
\]

What you have asked MAPLE to do is to solve the set of equations \(\{a_1, a_2, a_3\}\) for the variables \(\{i_1, i_2, i_3\}\), and that’s what you get.

If at this point, you would want to do the problem with different parameters, all you would have to do is change the parameter (resistance or voltage) in the corresponding line, and re-enter each line that contains that equation.

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**Can We Do This With Symbols?**

Funny you should ask. Yes indeed. Enter the equations

\[
>\text{b}_1:=r_1\text{I}_1+r_2\text{I}_2=e_1+e_2;
>\text{b}_2:=r_3\text{I}_3+r_2\text{I}_2=e_3+e_2;
>\text{b}_3:=\text{I}_1+\text{I}_3=\text{I}_2;
\]

where the currents are now identified by upper case letters and the resistances and voltages by lower case. It isn’t worth getting MAPLE to do a script “\(E\)”. Now, it’s just the same thing,

\[
>\text{solve}\{\text{b}_1,\text{b}_2,\text{b}_3\},\{\text{I}_1,\text{I}_2,\text{I}_3\};
\]

and by golly, it should work admirably.

If at this point you want numerical values, enter them in the lines

\[
>\text{r}_1:=140: \text{r}_2:=210: \text{r}_3:=35:
>\text{e}_1:=92: \text{e}_2:=55: \text{e}_3:=57:
\]

(the use of colons instead of semicolons just means that the values won’t be displayed). Now, repeat the solve command,

\[
>\text{solve}\{\text{b}_1,\text{b}_2,\text{b}_3\},\{\text{I}_1,\text{I}_2,\text{I}_3\};
\]

(those familiar with MAPLE know that instead of copying, you could go to that line and re-enter).
Who Want to Get Fancy?

Well, I do. For more involved problems, we would want our answers in a more usable form. Regarding the set of currents as a vector, we are solving a matrix equation, and ♣ can be written as

\[
\begin{bmatrix}
R_1 & R_2 & 0 \\
0 & R_2 & R_3 \\
1 & -1 & 1
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
= \begin{bmatrix}
\mathcal{E}_1 + \mathcal{E}_2 \\
\mathcal{E}_2 + \mathcal{E}_3 \\
0
\end{bmatrix}
\]

and solved by finding the inverse of the first (square) matrix on the left. This is the sort of thing that MAPLE does willingly.

First, the Linear Algebra package must be loaded into MAPLE with the command

\>`with(linalg);`

which shows you all of the thing you could now do. Perhaps you recognize some of the command names. If you are interested, go wild and call up the corresponding help window from the Help/Browser. Right now, all we need to do is create matrices, invert square matrices and multiply matrices. What we do is

\>`R:=matrix([[R1,R2,0],[0,R2,R3],[1,-1,1]]);`
\>`F:=matrix([[E1+E2],[E2+E3],[0]]);`

to get the proper form. Here, I use upper case for both voltages and resistances. As an interesting note, I can’t use \(E\) or \(I\) without a label, as these are reserved for the base of natural logarithms and the square root of \(-1\). So I chose \(F\), and lose points for originality.

To find the currents, the command is

\>`sols=multiply(inverse(R),F);`

where I use “sols” for “solutions”, mainly from habit. Compare to your previous results.