A Proper Finish to Class of September 29

What I had in mind was a geometric interpretation of the potential associated with a Coulombic field. The geometric interpretation *per se* is not crucial, but if you ever see, or wish to show to non-MIT folks, the needed expression without reliance on calculus, here you go.

My reference for this is *Contemporary College Physics* by E. R. Jones and R. L. Childers; the copy I have is the first edition (1990). To paraphrase from Page 169 of that text,

"That is,

\[ V_{AB} = \overline{E}_{AB} (r_B - r_A). \]

...Because of the inverse-square nature of the force, the appropriate average turns out to be the geometric average, or geometric mean. (This result can be shown with the aid of calculus.)"

Okay, at this point it’s clear that the J&C text will not be used at MIT. Still, let’s take on the responsibility of showing this result using calculus. That is, we know that

\[ V_{AB} = \frac{Q}{4\pi\epsilon_0} \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0} \left( \frac{1}{r_A} - \frac{1}{r_B} \right) = \frac{Q}{4\pi\epsilon_0} \frac{r_B - r_A}{r_A r_B}, \]

so that

\[ \overline{E}_{AB} = \frac{Q}{4\pi\epsilon_0} \frac{1}{r_A r_B}, \]

which is seen to be the geometric average of \( E(r_A) \) and \( E(r_B) \).

Whether or not this is useful physics is up to the observer. The above is certainly true. Consider, however, the possibility that you wish to show the above to someone who doesn’t know calculus, but is convinced that the energy done in moving an object against a force is the area under the force-distance graph. Then, you can break up the graph into a sum of small rectangles. If you take the height of each rectangle to be the geometric mean of the inverse square of the values of the positions at the boundaries of the rectangle, you will obtain a telescoping sum, yielding the desired result. You may have to read that sentence several times. A picture might help. The motivation for this math part comes from Simmons’s *Calculus with Analytic Geometry*, Problem *2* on Page 217.